

3.2 Estimation of Standing Timber, Growth and Cut

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3.2.1 Introduction

The basis for the standing timber estimation in the NFI are the individual tree volumes, which are estimated with the help of allometric equations. Tree stems are very different in their form. Estimations of the bole volume that are based only on the diameter at breast height $d_{1.3}$ are, therefore, considerably less precise than estimations that are also based on additional measurements of the tree height (H) and on an upper stem diameter. A higher estimation precision requires an augmented expenditure for the measurements.

Whereas the $d_{1.3}$ of all trees was measured in the NFI, an upper diameter (d_7 , diameter at 7 meter height) and the tree height were only measured on a subsample, the so-called tariff trees. The volume estimations of the tariff trees that were based on these three measurements ($Y_i = f(d_{1.3}, d_7, H)$) were used to derive tariff functions. The tariff functions estimate individual tree volumes depending only on one measurement variable (the $d_{1.3}$), but additionally on various tree, stand, and site attributes. To take three measurements from all trees for volume estimation is not efficient, since the gain in precision is small compared to the expenditure (KAUFMANN 1993; MANDALLAZ 1995; 1997).

In the first NFI, four trees per sample plot were measured on average (i.e. slightly more than one third of all trees) in a sector of 0 to 150 gon of the plots (ZINGG and BACHOFEN 1988). In the second NFI, on average two trees per plot were randomly chosen as tariff trees in this sector. The selection probability was proportional to $d_{1.3}^2$ (KAUFMANN 1993). For the timber assortment, according to the Swiss timber trade customs (Forstwirtschaftliche Zentralstelle 1976), tree stems are broken down into commercial assortments with the help of taper equations, which continuously describe a stem profile from the ground to the top.

3.2.2 Functions for the Estimation of Individual Tree Volumes

3.2.2.1 Bole Volume Functions

The bole volume functions of the NFI estimate the total overbark bole volume of a tree depending on the $d_{1.3}$, the d_7 and the tree height. Proportions of merchantable assortments are estimated with the help of taper equations (Chapter 3.2.6.1), which describe a stem profile depending on the same three variables. It is, of course, possible to estimate bole volumes with taper equations. Bole volume functions, which estimate the volume depending on the measured variables directly, are by far more simple and usually more precise than taper equations (BIGING 1984; KAUFMANN 1993). As the basis for the estimation of standing volume, growth, and cut both taper equations, as well as bole volume functions, were therefore developed for the NFI.

Measured variables

Since the three-parameter volume functions ($Y_i = f(d_{1.3}, \text{upper diameter}, H)$) are by far more precise than two-parameter functions ($Y_i = f(d_{1.3}, H)$), and since the precision requirement is very high in large-scale inventories in most of the European national inventories, the tree height and an upper diameter of at least some of the trees are measured. Stem forms can be precisely differentiated when a diameter in approximately 30% of the tree height (BRAUN 1969; KUBLIN and SCHARNAGL 1988; POLLANSCHÜTZ 1965) is measured. SCHMID-HAAS and WINZELER (1981) and WINZELER (1986) suggested that a measurement of an upper diameter at a height of 7 m with the use of the Finnish caliper (upper stem caliper), combined with a tree height measurement using the dendrometer "Christen", are most suitable when the expenditure for the

measurements, the expected measurement error and the estimation precision of three-parameter volume functions are all considered.

This measuring method, which was developed in the 1960's for the inventory of Swiss forest enterprises (SCHMID-HAAS *et al.* 1993) was, therefore, adopted for the NFI. Research conducted by WINZELER (1986) as well as the results from the NFI check assessment (Chapter 2.9) confirmed that the three variables can be measured without bias when using the instruments mentioned above.

Overbark bole volume functions

The following functions were derived mainly with the data from approximately 38,000 trees that were measured during the last decades in 2-meter sections in the course of the forest growth and yield studies at the WSL. Since these sample trees did not cover the whole range of stem forms found in the NFI, 500 additional trees (especially species such as spruce, larch, and beech) with extreme form ratios ($d_7/d_{1.3}$ and $H/d_{1.3}$) were measured.

$$\text{Spruce: } Y_i = b_0 + b_1 \cdot d_{7i}^2 H_i + b_2 \cdot d_{1.3i}^2 + b_3 \cdot d_{7i}^3 + b_4 \cdot H_i \quad (1)$$

$$\text{Fir: } Y_i = b_0 + b_1 \cdot d_{7i}^2 H_i + b_2 \cdot d_{1.3i} + b_3 \cdot d_{1.3i}^2 + b_4 \cdot d_{1.3i}^3 \cdot H_i + b_5 \cdot H_i^4 \quad (2)$$

$$\text{Pine: } Y_i = b_0 + b_1 \cdot d_{7i}^2 H_i + b_2 \cdot d_{1.3i} + b_3 \cdot d_{1.3i}^2 + b_4 \cdot d_{1.3i}^3 H_i \quad (3)$$

$$\text{Larch: } Y_i = b_0 + b_1 \cdot d_{7i}^2 H_i + b_2 \cdot d_{1.3i}^2 + b_3 \cdot H_i^2 \quad (4)$$

$$\text{Douglas fir: } Y_i = b_0 + b_1 \cdot d_{7i}^2 H_i + b_2 \cdot d_{1.3i}^2 + b_3 \cdot d_{1.3i}^3 + b_4 \cdot d_{1.3i}^2 \cdot H_i^2 \quad (5)$$

Coniferous trees (all species):

$$Y_i = b_0 + b_1 \cdot d_{7i}^2 H_i + b_2 \cdot d_{1.3i}^2 + b_3 \cdot d_{7i}^2 + b_4 \cdot d_{7i}^3 + b_5 \cdot d_{1.3i} H_i^3 \quad (6)$$

$$\text{Beech: } Y_i = b_0 + b_1 \cdot d_{7i}^2 H_i + b_2 \cdot d_{1.3i}^2 + b_3 \cdot d_{7i}^3 + b_4 \cdot d_{1.3i}^3 \cdot H_i \quad (7)$$

$$\text{Oak: } Y_i = b_0 + b_1 \cdot d_{7i}^2 H_i + b_2 \cdot d_{1.3i}^2 + b_3 \cdot d_{1.3i}^3 + b_4 \cdot d_{1.3i}^3 \cdot H_i \quad (8)$$

Deciduous trees (all species):

$$Y_i = b_0 + b_1 \cdot d_{7i}^2 H_i + b_2 \cdot d_{1.3i} + b_3 \cdot d_{1.3i}^2 + b_4 \cdot d_{7i}^2 \quad (9)$$

where:

Y_i : Bole volume including bark in m^3

$d_{1.3}$: Diameter at breast height in meters

d_7 : Diameter at 7 meter heights in meters

H : Tree height in meters

The coefficients of the functions are shown in Table 1. The regressor variables of the function for species spruce presented above correspond to a model by WINZELER (1986); those for species fir correspond to a model by HOFFMANN (1984). All regression coefficients of all functions are different from zero at the 95% level (t-statistics). The fewest possible number of regressor variables were included in the functions. All of these improved the model substantially.

Table 1. Coefficients of the bole volume functions.

	b0	b1	b2	b3	b4	b5
Spruce	0.029504	0.46756	2.43885	-5.74664	-0.001826	
Fir	0.039594	0.35832	-0.39142	3.75195	-0.013314	1.62E-07
Scotch Pine	0.055349	0.40341	-0.63535	4.84573	-0.10114	
Larch	-0.0173	0.36366	2.49123	0.000107		
Douglas fir	0.013166	0.35079	2.67531	-2.95083	0.001096	
Conifers	0.008486	0.5436	2.8898	-1.94043	-4.93601	1.33E-05
Beech	0.002542	0.39466	2.56612	-3.67034	0.03567	
Oak	-0.026759	0.31686	5.01484	-7.71408	0.19704	
Broadleaf	-0.021786	0.39992	0.28036	2.30656	-1.20368	

Validation of the Bole Volume Functions

The behavior of the functions, when one measured variable increases and both of the others remain unchanged, can be investigated with the help of partial differentials. The partial derivatives, with respect to $d_{1,3}$, d_7 , and H of all of the bole volume functions for all possible $d_{1,3}$ - d_7 - H combinations in the NFI were, apart from a few exceptions ($\partial v/\partial H$ for spruce and fir for trees with $d_7 < 6$ cm), all positive. The volume of a tree almost always increases, therefore, when a predictor variable increases, apart from some rare exceptions.

The functions are very sensitive to changes in the d_7 . Measurement errors of the d_7 influence the volume estimation more than the ones of the $d_{1,3}$. It seems reasonable to develop volume functions that are sensitive to those predictor variables which are measured most precisely. But, if in addition to the measurement errors the model precision (i.e. the random error of a volume function) is considered, it is obvious that a d_7 sensitive function estimates a bole volume more precisely than a $d_{1,3}$ sensitive function.

The reason for this is because the d_7 explains more variation than the $d_{1,3}$. This fact is shown in the following by comparing two volume functions for spruce (Functions 1 and 2). Function 1 (the function used in the NFI) is sensitive to changes of the d_7 . Function 2 reacts most sensitively to changes in $d_{1,3}$.

Function 1:

$$Y_i = 0.029504 + 0.46756 d_7^2 H + 2.43885 d_{1,3}^2 - 5.74664 d_7^3 - 0.0018265 H \quad (1)$$

Function 2:

$$Y_i = -0.052455 + 0.098718 d_{1,3}^2 H - 2.62032 d_{1,3} + 12.9122 d_{1,3} d_7 + 0.020145 H(10)$$

The residual variance components, which are due to the random measurement errors of the input variables, were estimated with a first order Taylor series expansion (KAUFMANN 1999). The random volume estimation error is composed of the measurement errors of the input variables and the model error (variance of the residuals), as it is shown in Figure 1. The variance components stemming from the measurement errors are the averages of 300 replications of stochastic simulations. The measurement errors used in the simulations correspond to those observed in the check assessment of the NFI (Chapter 2.9 WINZELER 1988).

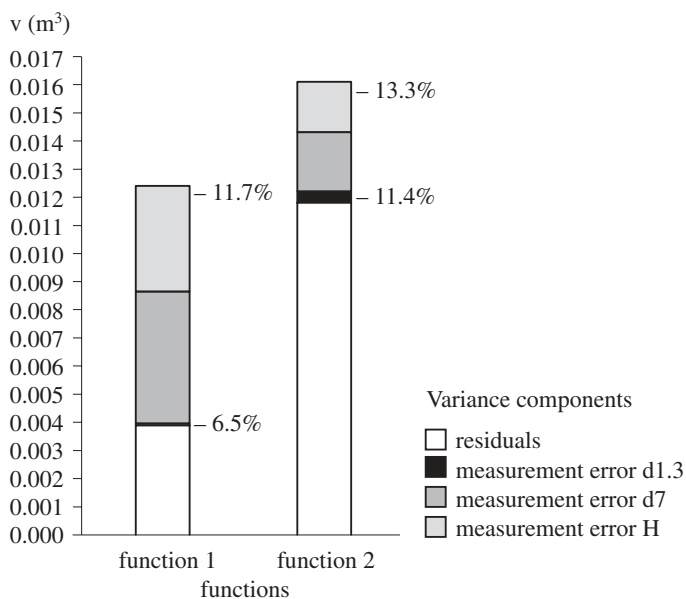


Figure 1. Error components (variances in m^3) of the bole volume functions 1 and 2 for spruce: measurement errors of $d_{1,3}$, d_7 and h as well as model prediction errors.

Due to the higher sensitivity of Function 1 to changes in d_7 , the d_7 -measurement error has a stronger affect here than in Function 2. The model error (i.e. the standard deviation of the residuals, of 11.4% of the mean volume for Function 2), is higher than the one for Function 1 with 6.5%. This is due to the fact that the $d_{1,3}$, which is not able to explain as much of the variation for the volume estimation as the d_7 , has a strong influence in Function 2. The total random error of 13.3% of the mean volume for Function 2 is, despite the smaller influence of the measurement error of the d_7 and the tree height, higher than for Function 1. Therefore, Function 1 gives more precise volume estimates than Function 2, even though the d_7 is measured with a higher random error than the $d_{1,3}$.

For all tree species, the d_7 is able to explain more of the variation for the stem volume than the $d_{1,3}$. Estimations of the standing timber with the d_7 sensitive NFI volume functions presented in chapter 3.2.2.1 are, therefore, more precise than with the $d_{1,3}$ sensitive functions. Using 1,000 trees, an estimation precision of 0.3% (standard error of the mean) was determined empirically (simulation with 50 replications) for the NFI volume functions, when random measurement errors, model errors and excess errors (see below) are taken into account.

How well the bole volume functions fit the sample tree data is presented in Table 2. The average residual (ar) in percentage of the mean volume (average deviation of the estimated volume with the help of a function from the volume calculated based on the diameter measurements in 2 m sections) is displayed in the upper portion of each cell. The standard deviation of the residuals (in percentage of the mean) is given within parentheses. The probability p_t of the t-distribution, that a mean estimated value deviates systematically from the true value, is shown in the middle of each cell. The bole volume functions explain a very large proportion of the individual tree volume variance (all coefficients of determination $R^2 > 98.5\%$).

Table 2. Goodness-of-fit of the bole volume functions: ar: average residual ($\Sigma(\hat{x}_i - x_i) / n$) and standard deviation of the residuals (rs, in parenthesis) in percentage of the mean. p_t : Probability value of the t-distribution. n: Number of trees. R^2 : Coefficient of determination.

		d1.3 (cm)									
		12.–23	24–35	36–47	48–59	60–71	72–83	84–95	96–107	total	
Spruce	ar (sr)	0.0 (3.61)	0.0 (3.61)	0.1 (3.95)	-0.2 (4.45)	-0.3 (5.08)	0.9 (3.84)	0.5 (3.55)	0.0 (3.79)	0.0 (6.53)	
	(R^2 : 99.7%)										
	p_t	0.55	0.75	0.31	0.2	0.38	0.07	----	----		0.88
	n	6622	3521	1780	779	253	60		3	13029	
Fir	ar (sr)	1.4 (5.00)	0 (4.58)	-0.1 (4.98)	-0.3 (4.90)	0.2 (4.91)	0.1 (5.18)	0.4 (4.46)	0.5 (3.89)	0.0 (7.15)	
	(R^2 : 99.6%)										
	p_t	0	0.67	0.48	0.12	0.44	0.79	0.47	0.55	0.7	
	n	1628	2199	1495	848	423	158	67	19	6848	
Scotch pine	ar (sr)	0.7 (4.74)	0.0 (3.92)	-0.4 (5.03)	1.1 (7.86)	-2.5	-1.4			-0.1 (6.16)	
	(R^2 : 99.3%)										
	p_t	0	0.92	0.13	0.45	----	----			0.73	
	n	487	789	334	31	2	1			1644	
Larch	ar (sr)	0.8 (4.72)	0.1 (3.87)	-0.2 (4.10)	0.0 (4.57)	0.2 (3.96)	-0.3 (6.47)	0.7 (12.02)	0.9	0.0 (6.15)	
	(R^2 : 99.4%)										
	p_t	0	0.64	0.28	0.98	0.56	0.8	----	----	0.98	
	n	303	454	494	232	102	26	5	1	1617	
Douglas Fir	ar (sr)	-0.8 (4.32)	-0.1 (4.21)	0.3 (3.83)	1.1 (3.15)	0.9 (4.11)	-1			-0.1 (4.76)	
	(R^2 : 99.7%)										
	p_t	0.01	0.69	0.35	0.1	----	----			0.47	
	n	169	260	108	23	5	2			567	
Conifers	ar (sr)	0.5 (4.40)	-0.3 (4.74)	0.1 (5.14)	0.2 (5.41)	0.3 (5.49)	-0.1 (5.87)	-0.9 (5.25)	-1.0 (4.26)	0.0 (8.17)	
	(R^2 : 99.5%)										
	p_t	0	0	0.29	0.07	0.2	0.85	0.14	0.27	0.54	
	n	9739	7686	4418	1955	794	248	83	23	24957	
Beech	ar (sr)	-0.1 (5.06)	-0.1 (5.69)	0.2 (7.43)	0.2 (9.24)	-0.4	-2.7 (9.50)			0.0 (11.29)	
	(R^2 : 98.7%)										
	p_t	0.19	0.34	0.43	0.68	0.79				0.91	
	n	2515	2188	863	244	46	7			5863	
Oak	ar (sr)	-0.9 (5.21)	0.5 (4.96)	-0.8 (5.31)	1.0 (7.09)	0.7 (4.92)	-0.1 (3.55)	-1		0.1 (7.80)	
	(R^2 : 99.5%)										
	p_t	0	0.02	0.02	0.17	0.4				0.71	
	n	621	578	254	98	37	9	2		1599	
Broadleaf	ar (sr)	0.0 (5.14)	-0.2 (5.70)	0.1 (7.28)	0.9 (8.95)	-0.9 (9.03)	-2.5 (6.82)	-4.6		0.0 (10.88)	
	(R^2 : 98.9%)										
	p_t	0.96	0.14	0.55	0.37	0.16				0.89	
	n	3424	2886	1242	397	89	16	2		8056	

All bole volume functions estimate the mean of the sample tree volume unbiased at a 95% level ($p_i > 0.05$). The standard deviation of the residuals ranges from 6.15% (larch) to 11.35% (beech) of the mean. For all 12 cm diameter classes that contain at least 30 sample trees, the maximum systematic deviation from the mean of the sample trees amounted to 1.4%. The average deviations in the range from -0.9 to $+1.4\%$, which in themselves are not very large, are not random ($p_i < 0.05$) for some functions in the lowest or second lowest diameter class. In these classes, the number of trees is very large.

The goodness-of-fit is by itself not a sufficient measure for the qualitative assessment of a function. During the validation, the behavior of the function should also be tested with data that were not used for its derivation. One way to validate is to set aside a portion of the available data for the function verification. For example, if only 50% of the data are used for the function derivation of the tree species spruce, and the function is then tested with the other part of the data, the standard deviation of the residuals increases slightly from 6.53% to 6.58% of the mean.

Another technique is the cross-validation procedure that repeatedly estimates the function parameters. For each simulation run, a group of trees are left out of the analysis and are used for the model validation. Each tree is randomly assigned to a group. With this, each tree is left out exactly once. The number of simulations equals the number of groups. The volume of each sample tree is predicted by a function which was derived without that tree. For the cross-validation of the bole volume functions, 5% of the trees were left out at each simulation run. The increase of the residual variance resulting from this – the so-called excess error – is very small for spruce (0.3%); between 1.8% and 2.2% for fir, pine, larch, and oak, slightly higher for beech (3.5%) and highest for Douglas fir (5.1%). With an excess error of this order, no considerable increase of the model error is expected when the functions are applied to independent data.

Individual tree volumes can be more precisely estimated with three variables, since stem form differences are easily distinguishable with the help of an upper diameter measurement. A study of the function for the tree species spruce, which is very prominent in all production regions and altitudes, showed that by using regional volume functions, the gain in precision was negligible. The **Root of the Mean Squared Error** (RMSE, that is the square root of the mean quadratic error) decreased only by about 2%, from 0.0841 m^3 to 0.0824 m^3 . For the analysis of covariance with classification by region or altitude, the partial F-values for the classification variables (regions) are many times smaller than for the weakest regressor variable. Consequently, regional functions were not derived.

3.2.2.2 Tariff Functions

For most of the NFI sample trees (82% in the second NFI) only the diameter at breast height $d_{1.3}$ was measured. For the estimation of standing timber and increments, so called tariff-functions were derived. These functions estimate the bole volume of a tree with the help of only one measured variable (i.e., the $d_{1.3}$). Similarly, the input variables d_7 and H for the taper equation (Chapter 3.2.6.1) were estimated with tariff functions that had the same form as the ones presented below.

Standard Model

Suitable models for tariff functions are power functions (higher order polynomial models) or exponential functions, such as the ones proposed by HOFFMANN (1982) or PARDÉ and BOUCHON (1988). The following basic form of an exponential model for the bole volume \hat{Y}_i of a single tree was developed by HOFFMANN (1982) for employment in Swiss enterprise inventories:

$$\hat{Y}_i = \exp(b_0 + b_1 * \ln(d_{1.3i}) + b_2 * \ln^4(d_{1.3i})) \quad (11)$$

The estimation of bole volumes for the entire country with the $d_{1,3}$ as the only explanatory variable in the model is not precise enough. For a function of this type, the standard deviation of the residuals amounts to 37.8% of the mean. It is reduced to 34.0% if a separate function is derived for each main tree species. A further reduction to 31.3% results if production regions are distinguished. This variation finally achieves 26.3% for the tariff functions developed during the NFI. In addition to the $d_{1,3}$, individual tree, stand and site attributes are used as explanatory variables here. The functions are presented in the following.

NFI Tariff Functions

The relevant explanatory variables for bole volume prediction were identified with the help of an analysis of covariance. For this, the tariff functions were linearized using a logarithmic transformation. The analysis of covariance can be regarded as a hybrid between linear regression and analysis of variance. Several different regression surfaces are compared with different intercepts and slopes for each continuous regressor variable (COOK and CAMPBELL 1979). The classifying attributes “tree species”, “production region”, “storey to which a tree belongs”, and “bifurcation of a stem” were found to have a significant influence (F-statistics, $P_F < 0.05$). For the continuous variables, significant influence was found for, apart from the $d_{1,3}$, the elevation above sea level, the site quality and the d_{dom} (see equation 12).

The following form of an exponential function, which was used in a similar way by WINZELER (1986) for estimating the standing volume in the first NFI, proved to be suitable.

$$\hat{Y}_{zk} = \exp(b_{0zi} + b_{1zi} \cdot \ln(d_{1,3k}) + b_{2zi} \cdot \ln^4(d_{1,3k}) + \sum_{j=3}^7 b_{jzi} \cdot B_{jk}) \quad (12)$$

Indices:

- z: Type of dependent variable (1..3)
- k: Individual tree
- i: Tariff number (1..30) (see Table 3)
- j: Additional individual tree and sample plot attributes (3..7)

Variables:

- \hat{Y}_{zk} : Individual tree variable to be predicted:
 - \hat{Y}_{1k} : Tariff volume (stem wood with bark)
 - \hat{Y}_{2k} : d_7
 - \hat{Y}_{3k} : Tree height

b_0 - b_7 : Model coefficients (Table 4 for bole volume at the time of the first NFI)

B: Additional individual tree and sample plot attributes:

- B_{3k} : Site quality (GWL: “Gesamtwuchsleistung”): Total increment in kilogram dry matter per hectare and year (see Chapter 3.1)
- B_{4k} : d_{dom} : Average $d_{1,3}$ of the hundred trees having the largest $d_{1,3}$ per hectare in cm
- B_{5k} : Bifurcation of a stem (1: yes / 0: no)
- B_{6k} : Elevation above sea level (m)
- B_{7k} : Storey to which a tree belongs (0: tree belonging to the upper storey/ 1: tree not belonging to the upper storey)

The coefficients b_0 – b_7 were estimated using non-linear regression (Gauss-Newton method, SAS 1990) with the help of the tariff tree volumes. These volumes were estimated with the bole volume functions (Equation 1–9). The coefficients of the tariff functions for the time of the first NFI can be found in Table 4. The goodness-of-fit for the tariff functions (average residual \bar{a} , p-value of the t-distribution p_t , coefficients of determination R^2 , for explanation see Table 2) are presented in Table 5.

Table 3. Tariff numbers in NFI for the bole volume, $d_{1.3}$, H, and increment functions.

Tariff number	Tree species	Production region
201	Spruce	Jura
202		Plateau
203		Prealps
204		Alps
205		Southern Alps
206	Fir	Jura
207		Plateau
208		Prealps
209		Alps / Southern Alps
210	Scotch Pine	Jura
211		Plateau
212		Prealps / Alps / Southern Alps
213		Jura / Plateau / Prealps / Alps
214	Larch	Southern Alps
215		Other conifers
216	Beech	Jura
217		Plateau
218		Prealps
219		Alps
220		Southern Alps
221	Oak (all species)	Plateau
222		Jura/Voralpen/Alpen/Alpen-Südseite
223	Sycamore maple / plane	Jura / Plateau
224		Prealps / Alps / Southern Alps
225	Ash	Plateau
226		Jura / Prealps / Alps / Southern Alps
227	Chestnut	
228	Other broadleaf	Jura/Plateau
229		Prealps / Alps
230		Southern Alps

Tariff curves shift with the increasing development stage of a stand, which means that a tree in a higher stage of development has a larger stem volume than a tree with the same $d_{1.3}$ in a lower development stage (Figure 2). The stage of development is defined by a d_{dom} range (ZINGG and BACHOFEN 1988). The development stage was included in the tariff functions as a continuous variable in the form of the d_{dom} , which was estimated with the help of the trees on a sample plot. Studies of the NFI data set have shown that a d_{dom} , which is estimated with the help of the sample trees, is a suitable measure for the development stage of a stand in which a sample plot is located.

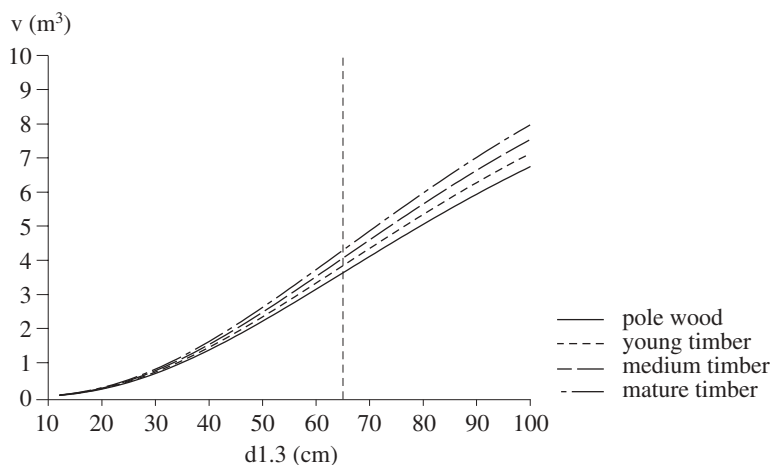


Figure 2. Tariff curves for species spruce in the alpine region for medium site quality in mountainous elevations, by development stage.

Tariff functions with $b_2 < 0$ (see Table 4) have an inflection point. This point is located at the culmination point of the volume increment. The reason for an inflection point is that the height growth of an individual tree culminates long before the $d_{1.3}$ increment and that trees with very large $d_{1.3}$ have, on average, a smaller form quotient ($d_7/d_{1.3}$) than the other trees. An inflection point prevents volumes of trees with a very large $d_{1.3}$ from being severely overestimated. Figure 3 shows two tariff curves (for spruce, upper storey, Alps, medium site quality, $d_{dom} = 55$ cm), which were derived with the data set of NFI tariff sample trees. One curve does not include the inflection point (i.e. without the term $\ln^4(d_{1.3})$), and another one includes the inflection point (i.e. with the term $\ln^4(d_{1.3})$ and $b_2 < 0$). The curve without this term increases very steeply in the upper diameter range, while the curve with an inflection point at 65 cm, flattens out and gives realistic volume estimates for the diameter range up to 200 cm.

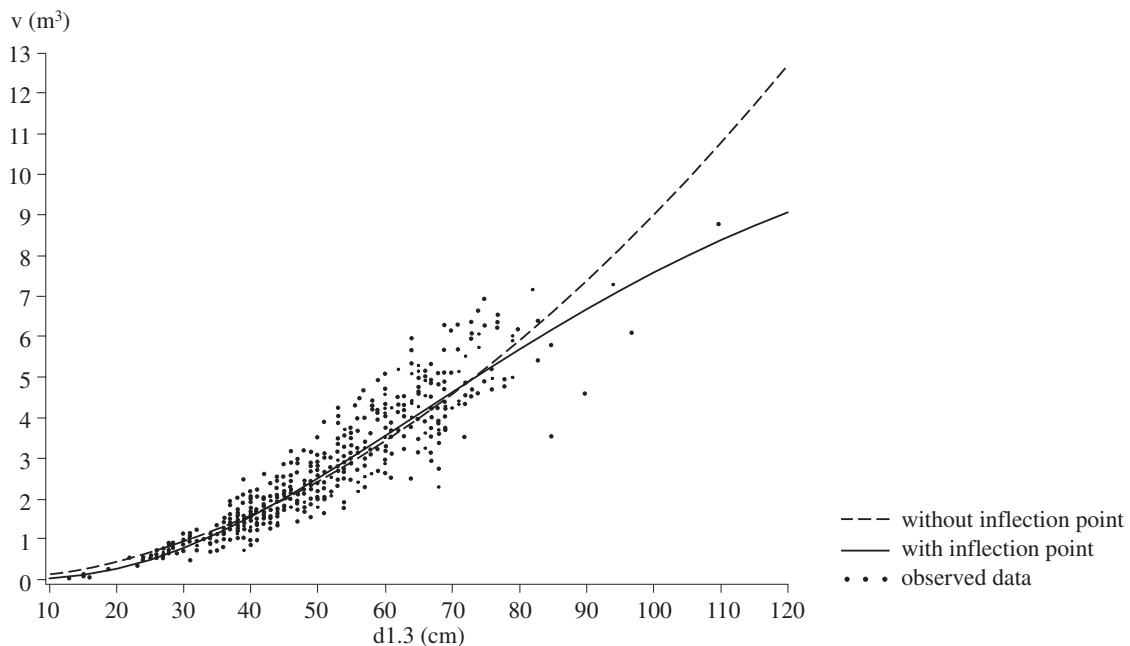


Figure 3. Tariff curves for spruce in the alpine region with and without inflection point.

When deriving the tariff functions, the fact was ignored that the sample trees were not selected completely independent from each other, but in clusters (sample plots). Since trees within one stand usually have similar forms, it is reasonable to assume that on a sample plot the residuals of the tariff functions are correlated with each other. This was indeed the case. Since the sample size in the NFI was large and the cluster sizes were small, the selection in clusters had no influence on the values of the function parameters as it was found in the study. The confidence intervals of the parameters, however, become slightly larger. BRIGGS and CUNIA (1982) came to the same conclusion in their study.

Table 4. Coefficients of the NFI1 tariff functions for the bole volume.

Tariff Number	b0	b1	b2	b3	b4	b5	b6	b7
201	-9.693932	2.875716	-0.003608	2.38E-05	0.006454	-0.354239	-0.000190	-0.293326
202	-10.19071	3.011815	-0.004360	5.66E-05	0.005186	0	-5.09E-05	-0.124890
203	-10.40762	3.148954	-0.004765	3.67E-05	0.005617	-0.292850	-0.000207	-0.345357
204	-11.22559	3.432392	-0.005889	3.39E-05	0.005502	-0.283506	-0.000226	-0.372618
205	-11.02461	3.208716	-0.005054	1.15564E	0.003814	-0.253676	-4.37E-05	-0.368829
206	-11.14165	3.390109	-0.005383	0	0.005589	-0.087306	-0.000157	-0.285632
207	-8.373972	2.428760	-0.002026	3.13E-05	0.005209	0	0	-0.266245
208	-9.127325	2.735734	-0.002371	2.53E-05	0	-0.221676	-5.74E-05	-0.228611
209	-13.29496	4.119043	-0.007279	3.69E-06	0.005529	-0.301906	-0.000352	0
210	-6.819805	1.795885	0.002160	1.97E-05	0.006278	-0.230662	-0.000452	-0.338596
211	-10.72603	3.230300	-0.004654	0	0.002566	0	0	0
212	-10.14396	2.772788	-0.003579	7.62E-05	0.013275	-0.157258	0	-0.309897
213	-10.99258	3.264363	-0.005054	7.14E-05	0.003558	-0.154775	-0.000124	0
214	-10.46496	3.186909	-0.004321	0	0.003074	-0.555413	-0.000304	0
215	-10.14301	3.161973	-0.004693	0	0.004242	-0.137419	-0.000482	-0.388137
216	-9.760576	2.838556	-0.003247	4.15E-05	0.006981	-0.190014	-0.000152	-0.397608
217	-10.86935	3.209637	-0.004539	5.83E-05	0.003261	-0.088868	0	-0.437208
218	-10.59635	3.112840	-0.004627	4.70E-05	0.008531	-0.289871	-0.000150	-0.278017
219	-11.03685	3.277674	-0.005875	3.35E-05	0.014177	-0.125906	-0.000395	-0.517794
220	-8.115184	2.171664	-0.000869	0	0.007345	-0.220946	0	-0.179818
221	-11.03194	3.359299	-0.005177	0	0.002811	0	-0.000202	0
222	-8.957069	2.503072	-0.002034	7.18E-05	0.007726	-0.347306	-0.000375	0
223	-8.015882	2.179587	-9.05E-05	5.85E-05	0.006317	-0.130022	-0.000225	-0.248277
224	-10.67774	3.402548	-0.007804	0	0.005672	-0.113936	-0.000456	-0.148527
225	-11.12480	3.271653	-0.005234	9.41E-05	0.004465	-0.257368	-0.000139	0
226	-8.661268	2.437937	-0.001629	5.90E-05	0.008327	-0.169188	-0.000342	-0.171477
227	-6.269070	1.663056	5.79063E	0	0	0	-0.000380	-0.333926
228	-9.020448	2.555589	-0.002027	4.39E-05	0.002062	-0.239662	0	-0.214907
229	-7.719545	1.867530	0.002000	5.24E-05	0.010988	-0.274791	0	-0.280542
230	-9.757205	3.338594	-0.006130	-0.000264	0	-0.341705	-0.000510	-0.234645

3.2.2.3 Error Propagation of Stem Volume and Tariff Functions

Random and systematic errors of the volume and tariff functions influence the variance and the expected values of inventory results. The extent of these influences was studied with Monte-Carlo simulations. Table 6 presents the results of a simulation study, which investigated the influences of random measurement errors of the input variables $d_{1,3}$, d_7 , and H , and of random prediction errors of the bole volume functions on the standard error of the total volume. The volume estimations are based on the approximately 11,000 sample plots of the first NFI. The generated measurement errors corresponded to those observed in the control measurements in the NFI (Chapter 2.9 WINZELER 1988) and the residuals of the bole volume functions to those presented in table 2.

On average, two tariff sample trees per plot were chosen with the selection method used in the second NFI (see Chapter 3.2.4). The volume was predicted once with the tariff volume (Table 6, columns A) and once with the “weighted residual” (WR) technique (Table 6, columns B, description of the expansion technique see Chapter 3.2.3). Furthermore, it was assumed that the residuals of the bole volume functions were independent of each other within one sample plot (Table 6, row 2) and, that these residuals were correlated with each other within one sample plot. That is to say, all generated residuals within one sample plot were positive or negative each time, but overall they did not differ from zero (Table 6, row 3).

Table 5. Goodness-of-fit of the tariff functions: ar: average residual ($\Sigma(\hat{X}_i - x_i) / n$) and standard deviation of the residuals (sr, in parenthesis) in percentage of the mean.
 p_t : Probability value of the t-distribution. n: Number of trees. R^2 : Coefficient of determination.

		d1.3								
		12.-23	24-35	36-47	48-59	60-71	72-83	84-95	96-107	total
Spruce (R^2 : 92.5%)	ar (sr)	-5.9 (27.7)	-0.7 (22.9)	0.3 (18.7)	-0.1 (18.6)	0.1 (18.9)	-1.1 (19.5)	4.4 (23.9)	-4.4 (24.9)	-0.2 (25.3)
	p_t	0	0.08	0.16	0.87	0.76	0.25	0.13	0.39	0.41
	n	4994	3682	6022	2683	1738	431	70	24	19647
Fir (R^2 : 95.7%)	ar (sr)	-1.8 (65.3)	-1.2 (20.3)	0.1 (16.5)	0.1 (16.1)	0.2 (14.1)	-0.3 (13.6)	-0.4 (13.3)	-5.2 (15.8)	-0.1 (21.5)
	p_t	0.25	0.05	0.75	0.89	0.75	0.81	0.89	0.46	0.7
	n	1781	1095	1612	648	497	154	28	6	5826
Scotch Pine (R^2 : 94.8%)	ar (sr)	4.2 (25.8)	-0.4 (20.1)	-1.0 (17.3)	1.7 (14.1)	-1.0 (15.9)	-2.7 (9.7)	5.7		0.0 (20.3)
	p_t	0	0.76	0.17	0.08	0.61	0.63	--		0.99
	n	448	309	573	202	61	4	1		1598
Larch (R^2 : 91.3%)	ar (sr)	2.6 (36.4)	0.1 (28.1)	0.4 (22.6)	-0.8 (21.5)	-0.2 (20.1)	-0.5 (22.9)	3.3 (16.8)	4.8 (17.9)	0.0 (27.6)
	p_t	0.12	0.94	0.63	0.51	0.87	0.83	0.36	0.51	0.99
	n	483	372	608	313	282	84	23	7	2174
Conifers (R^2 : 95.7%)	ar (sr)	-12.5 (37.5)	-12.8 (27.8)	-1.0 (25.1)	-0.5 (21.5)	1.4 (17.7)	-1.7 (14.3)	-0.5 (6.6)	-2.4 (17.3)	-1.5 (25.1)
	p_t	0	0	0.7	0.89	0.61	0.59	0.89	0.8	0.22
	n	124	81	99	45	39	22	4	4	420
Beech (R^2 : 94.6%)	ar (sr)	-8.5 (27.9)	-0.3 (23.6)	0.4 (19.3)	0.5 (17.5)	-1.2 (18.1)	-0.1 (17.9)	5.2 (19.8)	8.1 (25.2)	-0.6 (27.7)
	p_t	0	0.59	0.33	0.5	0.24	0.98	0.43	0.73	0.04
	n	3533	1853	1877	607	295	68	10	2	8245
Oak (R^2 : 95.5%)	ar (sr)	0.8	0.4	-3.1	2.2	1.7	-4.2	0.6	9.2	-0.2 (27.1)
	p_t	0.54	0.8	0.03	0.26	0.46	0.16	0.91	0.3	0.79
	n	405	186	183	82	68	29	9	3	965
Ash (R^2 : 94.7%)	ar (sr)	-0.2 (25.0)	-2.3 (19.3)	1.1 (19.3)	-1.0 (17.6)	-1.0 (16.9)	1.5 (23.6)			-0.4 (28.1)
	p_t	0.83	0.03	0.38	0.69	0.78	0.9			0.61
	n	741	322	225	55	25	5			1373
Maple (R^2 : 92.5%)	ar (sr)	1.7	-1	-1.1	5.8	-4.6	3.2			-0.1 (31.8)
	p_t	0.06	0.34	0.49	0.36	0.4	0.62			0.93
	n	735	328	161	30	13	5			1272
Chestnut (R^2 : 88.8%)	ar (sr)	22.5 (26.4)	-4.7 (26.0)	-12.5 (27.5)	-4.5 (29.2)	-3.4 (32.3)	1.5 (33.2)	2.4 (37.2)	11.3 (35.4)	1.3 (62.1)
	p_t	0	0.09	0	0.43	0.61	0.82	0.83	0.34	0.57
	n	459	91	61	28	25	25	12	10	719
Other broadleaf (R^2 : 94.3%)	ar (sr)	1.0 (27.6)	-0.4 (25.2)	0.2 (25.2)	0.4 (22.2)	-3.9 (20.0)	10.4 (17.2)	17.4	-4.5 (9.6)	0.2 (39.4)
	p_t	0.14	0.79	0.92	0.91	0.34	0.25	--	--	0.83
	n	1583	352	161	42	25	5	1	2	2171

Table 6 demonstrates that the variance between the sample plots is influenced only slightly by the random error of the bole volume functions. The variance increased by only 2.2% (the standard error of the volume by 1.1% respectively) for the worst variant B and the error assumption 2 in Table 6 as compared to the variance, which was determined without considering random error in single tree volume estimation. Similarly, GERTNER and KÖHL (1992) also found that the random error of the three-parameter volume function only slightly affected the standard error of the standing timber in the NFI.

When the WR technique (Chapter 3.2.3) is used, the standard error component caused by the random errors of the tariff functions is completely included in the sampling error. This is not the case when volume is estimated only with the help of tariff functions. The standard error for the total volume increases by 6.9% compared to the volume estimation only with tariff functions when the WR technique is used and random model errors of the bole volume functions are also considered (Table 7, row 2). The standard error increases by 7.3% when, in addition to the WR-technique, random measurement errors are included (Table 7, row 3). It also finally increases by

7.8% when it is additionally assumed that the residuals of the tariff functions within a plot are highly correlated with each other (Table 7, row 4).

In contrast to the effects of the random error components for the individual tree volume estimation, the effects of the systematic errors do not decrease with increasing sample sizes. WINZELER (1986) attached more importance to the systematic measurement error than to the random ones. GERTNER and KÖHL (1992) determined similarly in their study with the tree species spruce that the NFI method of volume estimation is very sensitive with respect to systematic measurement error.

The resulting biases (in percentage of the total volume) of the simulation study (with 10 replications) described above are presented in Figure 4. A systematic error was added here to the individual tree measurements. A systematic measurement error of +0.5 cm for the $d_{1,3}$ led to a volume overestimation of 1.0%. A measurement error of +0.5 cm for the d_7 led to an overestimation of 2.1%, and a tree height measurement error of +0.5 m resulted in an overestimation of 1.6%. Employing the same systematic measurement error, the upper diameter caused a double bias in comparison to the $d_{1,3}$.

Table 6. Propagation of random errors of the NFI bole volume functions. Variances between sample plots (v) and corresponding standard errors (s), caused by random measurement and model prediction errors.

Total volume NFI1, random selection of two tariff trees per plot.

Method A: Expansion only with the tariff volumes. Method B: Expansion with the WR technique.

Errors considered:

1. Sampling error
2. Sampling error, random measurement and model prediction errors.
3. Sampling error, random measurement and model prediction errors assuming a maximal correlation of the residuals of the bole volume function within a plot.

Errors considered	A		B	
	v (%)	s (%)	v (%)	s (%)
1	100.0	100.0	100.0	100.0
2	100.2	100.1	101.0	100.5
3	100.6	100.3	102.2	101.1

Table 7. Propagation of random errors of the NFI tariff functions. Variances between sample plots (v) and corresponding standard error (s), caused by random measurement and model prediction errors. Total volume NFI1, random selection of two tariff trees per plot.

Errors considered:

Sampling error.

1. Sampling error
2. Sampling error, random model prediction errors.
3. Sampling error, random measurement and model prediction errors.
4. Sampling error, random measurement and model prediction errors assuming a maximal correlation of the bole volume function residuals within a plot.

Errors considered	v (%)	s (%)
1	100.0	100.0
2	114.3	106.9
3	116.0	107.3
4	116.1	107.8

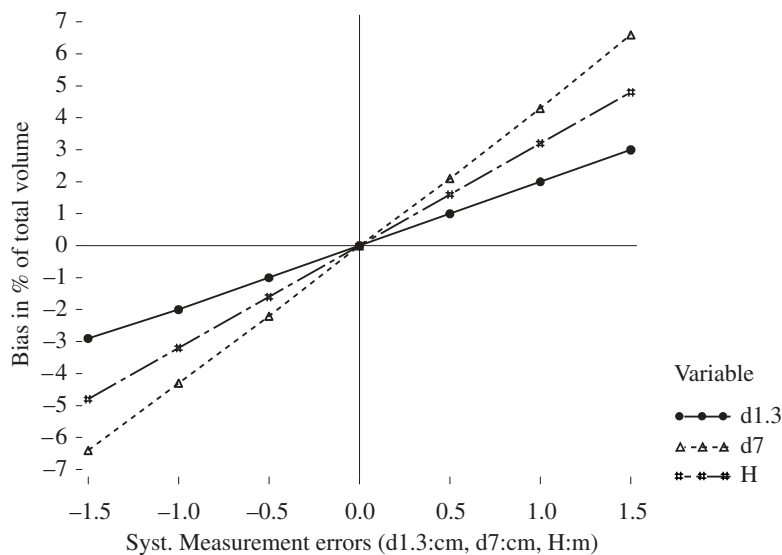


Figure 4. Estimated bias as a result of systematic measurement errors, in percentage of the NFI1 total volume.

3.2.2.4 Functions for Estimating Merchantable Branches and Slash

Merchantable timber includes, apart from the stem, the volume of large branches (with a diameter of at least 7 cm). The proportion (p) of large branch volume (a) to the stem volume over bark (v) is estimated with the help of a logit regression model. The functions were derived with the help of sample tree data from forest yield research plots. The branches of these 12,000 trees were measured.

The ratios $p_i = a_i/v_i$ are estimated using the following logit models:

$$\ln\left(\frac{p_i}{1 - p_i}\right) = b_0 + b_1 \cdot d_{1.3i} + b_2 \cdot h1_i + b_3 \cdot h2_i = \text{logit}(p_i) \tag{13}$$

and

$$p_i = \frac{\exp(\text{logit}(p_i))}{1 + \exp(\text{logit}(p_i))} \tag{14}$$

where:

b_0 – b_3 : Regression parameters (Table 8)

$d_{1.3}$: Diameter at breast height in centimeters

$h1, h2$: Indicator variable for elevation above sea level
 Region Alps: $h1 = 1$ for sites between 1000–1500 meters above sea level
 Other regions: $h1 = 1$ for sites between 600–1250 meters above sea level
 Otherwise $h1 = 0$

Region Alp: $h2 = 1$ for sites above 1500 meters above sea level
 Other regions: $h2 = 1$ for sites above 1250 meters above sea level
 Otherwise $h2 = 0$

The proportion of large branches for the species spruce is negligibly small, so the NFI database does not include any large branch volume for spruce. For the remaining conifers and broadleaf species (apart from beech), the large branch proportion is estimated based only on the $d_{1,3}$ (Table 8). For the tree species beech, the regression coefficients additionally differ significantly (likelihood ratio, see below) for the productive regions and for elevation. The significance of parameters was tested with the likelihood ratio test (p. 38, LINDER and BERCHTOLD 1982). The test statistic follows a chi-square distribution. For beech and the other broadleaf species, all parameters are significant ($P\chi^2 < 0.05$). The effect of the $d_{1,3}$ is not significant for fir ($P\chi^2 = 0.06$) and clearly not significant ($P\chi^2 > 0.05$) for the tree species larch and pine. This means that above a certain diameter, the volume for the large branches increases proportionally with the stem volume.

Table 8. Coefficients of the functions for merchantable branch and slash.

Regions	Species	b0	b1	b2
All regions	Fir	-8.7330758	0.059208154	0
All regions	Larch	-5.8871184	0.010812163	0
All regions	Scotch Pine	-7.7147742	0.072285665	0
Jura	Beech	-4.8322966	0.056314711	0
Plateau	Beech	-5.9903924	0.101889094	0
Prealps, Alps and				
Southern Alps	Beech	-4.9853383	0.073941728	-0.7056977
All Regions	Other broadleaf	-4.9398872	0.061619224	0

3.2.2.5 Growth Function

Growth functions were needed to estimate the volume of the so called ongrowth trees during the first NFI. Ongoing trees are the trees that reached the caliper threshold of 35 cm between the two inventories, and were measured only during the second inventory occasion (see Chapter 3.2.5). The cut trees were only recorded during the first inventory occasion; therefore, growth functions were also needed to estimate the increment of the cut trees for the half inventory interval. These functions were an important basis for the scenario models (see Chapter 3.3). A growth function was derived from the inventory data which either predicts a decadal basal area increment of a tree starting from the time of the first inventory occasion, or a corresponding decrement starting from the time of the second inventory occasion (KAUFMANN 1996):

$$\pm \text{BAI} = v_z \cdot \exp\left[b_0 + \sum_{j=1}^6 b_j \cdot B_{ji} + b_7 \cdot (v_z + 1) \cdot (1 - \exp(b_8 \cdot d_{1,3i})) + \dots\right. \\ \left. \dots + b_9 \cdot (v_z - 1) \cdot (1 - \exp(b_{10} \cdot d_{1,3i}))\right] \quad (15)$$

$$\pm \text{BAI} = \pi/4 (d_{1,3(2)}^2 - d_{1,3(1)}^2): \text{Basal area change in 10 years in m}^2$$

$d_{1,3(2)}$: $d_{1,3}$ at the time of the second inventory occasion

$d_{1,3(1)}$: $d_{1,3}$ at the time of the first inventory occasion

v_z : +1 for the prediction of basal area increment in the next 10 years

-1 for the prediction of basal area decrement in the previous 10 years

B_1 : BASFPH: Basal area per ha in m^2

B_2 : BAL: Basal area of all trees on a sample plot with a larger $d_{1,3}$ than the actually considered tree

BAL is a measure for the competition within a stand

B_3 : Site quality (GWL, see Chapter 3.1, 3.2.2.2)

B_4 : Elevation above sea level in meters

B_5 : Estimated stand age

B_6 : Storey membership of a tree (see Chapter 3.2.2.2)

b_0 – b_{10} : Model parameters with $0 \leq b_8 \leq 1$ and $0 \leq b_{10} \leq 1$ (Table 9)

The form of this growth model is similar to the ones developed by QUICKE *et al.* (1994) and TECK and HILT (1991). These models also use the competition factor BAL and express the basal area increment as a negative exponential function based on the $d_{1,3}$ ($BAI = b_7 \cdot (1 - \exp(b_8 \cdot d_{1,3}))$).

The precision of the growth function for the BAI is low compared to the tariff functions or to the stem volume functions. The R^2 for the tree species spruce, fir, and beech ranges between 40% and 60% for the Plateau, and between 30% and 40% for the Alps. The random measurement error of the $d_{1,3}$, assessment uncertainties of influence factors, and the heterogeneity of stand structures and sites were all too large to explain a diameter change precisely, the more so as a 10 years increment is relatively small. For example, in the Alps the 10 years increment was, on average, 2.8 cm for trees of the upper storey. Nevertheless, the precision of these functions was sufficient for the $d_{1,3}$ extrapolation of cut and ongrowth trees.

For unbiased estimates, bole volume ($v = f(d_{1,3}, d_7, H)$) of ongrowth and cut tariff trees (Chapter 3.2.5) must be extrapolated for the use of the WR technique (Chapter 3.2.3). Because of this, a growth function with the same formulation as the function described above, but with bole volume differences instead of basal area increment as dependent variable, was derived from the surviving tariff trees.

Table 9. Coefficients of the basal area – increment function for the uniform high forest.

Tariff Nr	b0	b1	b2	b3	b4	b5	b6	b7	b8	b9	b10
201	-5.972557	0	-0.01411773	1.23803E-04	-0.00014326	0	-0.44190505	1.21888055	-0.04126653	-1.5231641	-0.02379006
202	-5.3576716	-0.00253504	-0.01117313	1.26891E-04	-0.00012378	-0.00809598	0	1.6531399	-0.03091997	-2.1006859	-0.01937307
203	-5.5999909	-0.00430994	-0.0031308	4.65E-05	0	-0.00615716	-0.22077679	2.00165397	-0.02687856	-2.5092794	-0.01758865
204	-5.9825582	-0.00372182	0	3.80E-05	-0.0003579	0	-0.1401818	1.78000818	-0.03350251	-2.1295719	-0.02207817
206	-6.8444118	0	-0.00936935	5.95E-05	-0.00029545	0	0	1.91662896	-0.05240991	-2.2333833	-0.03183865
207	-5.447667	-0.00591097	0	0	2.31573E-04	-0.01053092	0	2.28000974	-0.03124656	-3.0428097	-0.01735486
208	-6.440883	-0.00284107	0	0	0	-0.00606455	0	2.36330484	-0.03903922	-2.830217	-0.02440044
209	-6.5055495	0	0	0	0	-0.01016251	0	2.56963065	-0.03892991	-2.7872422	-0.03099751
210	-5.8575938	0	-0.0121997	0	-0.00098791	0.011110966	0	0.781433661	-0.03031232	-10	-0.00124574
211	-6.9563871	0.007066096	-0.03209735	5.11243E-04	0	0.006015048	0	-0.01971149	-0.09852732	0.086656529	0
212	-6.426157	-0.00741484	0	0	0	0	-0.0001	1.65054997	-0.02778707	-2.2765818	-0.01636538
213	-6.795361	-0.005249	-0.01031298	1.33487E-04	0	0	0	1.67724725	-0.0509718	-1.8539024	-0.035385
215	-6.2673073	0	0	0	0	0	0	1.27544562	-0.06649714	-3.1697299	-0.01099584
216	-6.0035754	-0.01002982	0	5.92E-05	0	-0.00608263	0	1.98923478	-0.03618071	-2.3349154	-0.02527642
217	-7.5435469	0	-0.00548712	1.36378E-04	-0.00032369	0	0	2.17634327	-0.05583974	-2.3437736	-0.03913801
218	-6.9790127	-0.00329626	0	4.21E-05	-0.00050808	0	0	2.20321716	-0.04875115	-2.5636204	-0.03059659
219	-6.5431542	0	-0.01247515	7.35E-05	-0.00070747	0.003120768	-0.54523011	1.55170032	-0.07531738	-1.5899548	-0.05562687
221	-6.9921994	0.008601155	-0.00953882	1.77502E-04	0	0	0	1.37321591	-0.04713288	-1.5589308	-0.03041591
222	-6.1045743	0	0	0	0	0	0	3.60419634	-0.00836617	-7.0902276	-0.00356603
223	-6.5274475	0	-0.01722707	9.07E-05	0	0	0	1.31721666	-0.07515075	-1.4562757	-0.05199407
224	-5.8086525	-0.00885217	0	1.08701E-04	0	0	0	3.10339974	-0.00754785	-10	-0.00208437
225	-5.7187214	0	0	0	0	0	0	1.54015364	-0.03632488	-3.9809956	-0.00779767
226	-6.4276149	0	0	5.21E-05	0	-0.00809001	0	2.2956149	-0.03516852	-2.5650228	-0.02689375
227	-23.117758	0	0	0	0	0	0	9.40725503	-0.17709636	-10	-0.0964788
228	-6.3296482	0	0	0	0	0	0	3.84330169	-0.01163934	-4.6639498	-0.0082585
229	-7.8887806	-0.01893619	0	0	-0.00049786	0	0	2.8269974	-0.06503545	-3.0844061	-0.04931764

3.2.2.6 Stand Age Prediction

The stand age of uniform forests is used as an input variable in the growth functions. Tree ring counts on stumps of freshly cut stems on the sample plots were relevant here (STIERLIN *et al.* 1994). As the stand age could be determined this way only for a portion of the sample plots, a regression function was derived, which allowed predicting the age of a stand in a uniform forest based on explanatory variables that were assessed on all of the sample plots.

$$\text{Age}_i = \exp\left(b_0 + \sum_{j=1}^3 b_j B_{ji}\right) \quad (16)$$

B_1 : $d_{\text{dom}}/\ln(\text{GWL})$ (definition d_{dom} and GWL see Chapter 3.2.2.2)

B_2 : Elevation above sea level in meters

B_3 : Z / K

With:

Z : Increment of the survivor trees (pp. 308, HUSCH *et al.* 1972) in m^3/ha

K : Mean basal area diameter (p. 148, PRODAN 1965)

The coefficients b_0 – b_3 can be found in Table 10. The coefficient of determination R^2 of this function is 39% for the regions Jura and Pre-Alps, 54% for the Plateau, and 31% for the Alps.

Table 10. Coefficients of the function used to estimate stand age.

Region	b0	b1	b2	b3
Jura	4.054784	0.095935	3.90156E	-0.081179
Plateau	3.440956	0.188870	2.15274E	-0.057807
Prealps	3.945205	0.127037	1.64682E	-0.060118
Alps and Southern Alps	4.293609	0.061116	3.41993E	-0.078227

3.2.3 Expanding Individual Tree Volumes to Sample Plot Values

The $d_{1,3}$ was measured on all trees at both inventory occasions. From these trees a subsample of so-called tariff trees was selected on which, in addition, the d_7 and the tree height were measured. The volumes of these trees were estimated with the bole volume functions (Chapter 3.2.2.1). These volumes were not only used for the derivation of tariff functions (Chapter 3.2.2.2) but for a statistically correct expansion of single tree volumes to sample plot values as well. This was possible because the selection probability of the tariff trees was exactly defined.

In the first NFI a large number of tariff trees were selected. From these data, tariff functions were developed that had a solid foundation and that allowed unbiased volume estimates for large areas. Hence, it would not have been necessary to measure tariff trees in the second inventory. It would have been possible to estimate standing timber, increment, and cut with the help of the tariff functions that have been derived from the first NFI tariff trees. However, if the bole volumes are estimated only with the help of tariff functions, the results on the one hand could be biased for small sampling units; on the other hand, the variance between sample plots could be underestimated due to the smoothing effect of the tariff functions. MANDALLAZ (1991; 1997) and SÄRNDAL (1980; 1989) show that the “weighted residual” technique (WR), an expansion technique that uses the residuals of the tariff functions, results in unbiased estimates of standing volume even for small sampling units, if it can be assumed that the volumes calculated by the bole volume functions are unbiased. In addition, the variance smoothed out by the tariff functions is completely included in the standard error.

The residuals of the tariff functions that were used for the expansion were only known for the subsample of the tariff trees. The expansion to volumes per area unit, they are weighted proportionally to the inverse of the selection probability of the tariff trees (Horvitz-Thompson-estimator, cited in SÄRNDAL 1980). The estimator for the standing volume and the standard error of the WR technique are defined as follows:

Standing volume (per ha) of a sample plot x :

$$\hat{Y}(x) = \sum_{i \in S1(x)} f_i \cdot \hat{Y}_i + \sum_{i \in S2(x)} f_i \cdot \frac{\varepsilon_i}{\pi_i} \quad (17)$$

Expected value of the mean standing volume per sample plot:

$$\hat{Y} = \frac{1}{n} \sum_{x=1}^n \hat{Y}(x) \quad (18)$$

Standard error of the standing volume:

$$\hat{V}(\hat{Y}) = \frac{1}{n(n-1)} \sum_{x=1}^n (\hat{Y}(x) - \hat{Y})^2 \quad (19)$$

where:

\hat{Y}_i : Tariff volume of an individual tree i

$\hat{Y}(x)$: Standing volume of a sample plot x

\hat{Y} : Expected value (standing volume per hectare) estimated with tariff functions

$e_i = Y_i - \hat{Y}_i$: Residual of the tariff function

with Y_i : Individual tree volume estimated with a three-parameter bole volume function

($v = f(d_{1.3}, d_7, H)$)

S1(x): All trees on sample plot x

S2(x): Subsample of the tariff sample trees on sample plot x

f_i : Expansion factor to values per hectare (in NFI $f_i = \frac{10000 m^2}{200 m^2} = 50$ for trees with $d_{1.3} \leq 35$

cm, $f_i = \frac{10000 m^2}{500 m^2} = 20$ for trees with $d_{1.3} > 35$ cm)

p_i : Probability that a tree is selected as a tariff tree and that all three variables ($d_{1.3}$, d_7 , H) can be measured

$P_i = P_{selection} \cdot P_{measurable} \cdot P_{sector}$

$P_{selection}$: Selection probability of a tariff tree (see Chapter 3.2.4, Equation 25)

$P_{measurable}$ Probability that $d_{1.3}$, d_7 , and H are measurable

$P_{sector} = \begin{cases} \frac{150}{400} & \text{for trees with } d_{1.3} < 60 \text{ cm} \\ \frac{1}{1} & \text{for trees with } d_{1.3} \geq 60 \text{ cm} \end{cases}$

The expected values for Equation 18 are unbiased if the bole volume functions are unbiased. Equation 19 corresponds to the classical equation for standard error estimation of a one-phase simple random sample. This also means that the equations used for the standard error estimation of the double sampling (KÖHL 1994) are still valid when the WR technique is applied.

The expansion to standing volume per hectare for each sample plot ($\hat{Y}(x)$) is carried out with the Equation (17).

For the total standing volume of the terrestrial sample of the first NFI, the standard error is 0.6864% when the volume is estimated using only the tariff function values. It increases to 0.713% if the WR technique is used, which represents a variance increase between sample plots of 8.7%. The second standard error is larger, since it includes the random error of the tariff functions. Due to the large proportion of tariff trees (34% of all sample trees), the two variances differ only slightly.

Because tariff curves change with increasing stand age (PARDÉ and BOUCHON 1988) or due to silvicultural treatments, they can be biased after a few years, even if they are applied to the

same population they were originally derived from. Because of this, separate tariffs were derived for each inventory cycle from the tariff trees measured in each cycle. DUPLAT and PERROTTE (1981) also recommended this procedure. Nevertheless, a small bias of the tariff functions can result in a considerable bias of the increment estimation.

Suppose, for example, that the growth rate amounts to 2% per year (which corresponds approximately to the growth rate in the Alps), and that the increment is estimated with two tariffs. If one of the tariffs underestimates the standing volume by 2% and the other overestimates it by 2%, then the estimated increment is biased by 20%. Compared to this, the standard error of the increment amounts to 3% in the Alps.

In order to avoid systematic errors, the increment (see Chapter 3.2.5) was estimated with the WR technique in the following way:

$$\begin{aligned}
 G_g &= V_{sc2} - V_{sc1} = \hat{Y}_2(x) - \hat{Y}_1(x) \\
 &= \sum_{i \in S_1(x)} f_{2i} \hat{Y}_{2i} + \sum_{i \in S_2(x)} f_{2i} \frac{e_{2i}}{p_{2i}} \sum_{j \in S_1(x)} f_{2j} \hat{Y}_{1j} + \sum_{i \in S_2(x)} f_{2i} \frac{e_{1i}}{p_{2i}} \\
 &= \sum_{i \in S_1(x)} f_i (\hat{Y}_{2i} - \hat{Y}_{1i}) + \sum_{i \in S_2(x)} f_{2i} \frac{(e_{2i} - e_{1i})}{p_{2i}}
 \end{aligned} \tag{21}$$

i: Index for an individual tree

Index value 1 holds for first, value 2 for the second inventory

V_{sc1}, V_{sc2} : see Chapter 3.2.5, Equation 26 $S_1(x)$: all trees on sample plot x

$S_2(x)$: Tariff trees on the sample plots that were assessed in both inventories

f_{2i} : Expansion factor for inventory 2

p_{2j} : Selection probability for a tariff tree at inventory 2

e_{2i}, e_{1i} : Tariff function residuals at inventories 2 and 1 respectively of those tariff trees that were assessed and completely measured at both inventories.

The increment including the ingrowth G_{gi} (see Chapter 3.2.5) contains, in addition to G_g (Equation 21), the ingrowth calculated according to Equation 17.

The above mentioned properties of the WR technique (correction of biased tariff functions, including the random error of the tariff functions in the standard error) are illustrated with the following examples:

The 10 years increment was estimated with a subsample of 6,000 tariff trees that were assessed at both inventories (Table 11). The increment of these tariff trees estimated with the bole volume functions is assumed to be the true increment. For these trees, two different tariff functions were then applied for the two inventories: once the increment was calculated with the NFI tariffs (tariffs from the first NFI) and once with biased Lucerne tariffs (Tariffs II and III, p. 193 PFEIFFER 1993).

The volume increment determined by using the Lucerne tariffs was larger by a factor of 1.7 than the one estimated with the NFI tariffs. From all sample trees, a $d_{1,3}^2$ -proportional subsample was randomly selected using a sampling fraction of 18%, 9%, and 4.5%. Only the trees in this subsample were now regarded as tariff trees. The increment was estimated with the WR technique. This was repeated 100 times for each of the sampling fractions of tariff trees. The volume increment calculated using the NFI tariffs as well as the Lucerne tariffs were, on average, the same as those determined with the bole volume functions.

Table 11. Increment estimation using 6000 tariff trees for which in the first and second NFI the d_7 and the tree height were measured. Trees, which were actually treated for the expansion as tariff trees (18% of all trees), were selected proportionally to the d_{13}^2 . The simulation was repeated 100 times.

DV: average volume difference of the 100 simulation runs. The standard deviation of the volume differences are given in parenthesis.

Method of volume estimation	Proportion of tariff trees	Volume (Mio m ³) 1985	Volume (Mio m ³) 1995	DV (Mio m ³) (Mio m ³) 1985–1995
Volume functions	100%	285.3	361.3	76
Tariffs NFI	0%	280.1	354.1	74
Tariffs Lucerne	0%	261.1	386.4	125.3
WR-technique (volume functions and tariffs NFI)	18% 9% 4.50%	285.2 284.9 285.3	361.2 361.1 361.7	76.0 (1.5) 76.2 (2.0) 76.3 (3.3)
WR-technique (volume functions and tariffs Lucerne)	18% 9% 4.50%	285.4 285.2 285.8	361.4 361.3 361.5	75.9 (2.4) 76.1 (3.0) 75.7 (5.0)

The random errors shown in Table 11 are empirical standard deviations of the increment estimates in 100 simulation runs. They do not contain the sampling error. The deviations between the simulation runs are relatively small compared to the bias of the tariffs.

Table 12 shows the increment estimates of the survivor trees (trees measured at both inventory occasions) of the NFI sample plots located on a 100 km wide strip in the North-South direction throughout all of Switzerland.

Table 12. Increment estimation in a 100 km wide strip in a North-South direction through Switzerland. Using the tariff NFI1, the tariff NFI1 and NFI2, applying the WR technique.

A: Tariff volume differences; B: Increments estimated by using the WR technique.

Region	Number of plots	Number of tariffs used	A Mio m ³	B Mio m ³
Jura	328	1	7.3 (± 0.25)	7.7 (± 0.30)
		2	8.2 (± 0.26)	7.9 (± 0.31)
Plateau	539	1	10.5 (± 0.28)	10.6 (± 0.32)
		2	10.9 (± 0.29)	10.7 (± 0.34)
Prealps	506	1	9.0 (± 0.28)	9.3 (± 0.32)
		2	9.8 (± 0.29)	9.4 (± 0.31)
Alps	505	1	5.4 (± 0.24)	5.7 (± 0.28)
		2	7.0 (± 0.27)	6.0 (± 0.30)

The increments in column A are tariff volume differences: once using one tariff for both inventory dates (row 1 — derived from the tariff trees of the first NFI) and once using two different tariffs (row 2 — derived from the tariff sample trees of the first and second NFI respectively).

Column B shows the increments that were estimated using the WR technique. The increments in row 1/column A are systematically different from those in row 2/column B. The increment differences between row 1 and row 2 in column B are random. Furthermore, this table demonstrates, that the sampling error in column B is larger than the one in column A, since in column B the random errors of the tariff functions are included in the sampling error.

3.2.4 Tariff Tree Selection

In order to be able to estimate the standing volume, increment and cut with the WR-technique discussed in Chapter 3.2.4, the tariff trees had to be selected for each inventory period. Measuring the d_7 and H on all trees would have been inefficient, since the gain in precision

would have been relatively small compared to the additional arising expenses. It was, therefore, advantageous to select a subset of sample trees that achieved an optimal balance between expenditure and precision. For this optimization, the selection procedure played an important role.

Predicting the Variance between Sample Plots when all Sample Trees are Selected as Tariff Trees

The variance of interest is the one that could have been expected, if in the first NFI the d_7 and the tree height of all trees would have been measured and the standing volume would have been estimated with the bole volume functions. This variance can be estimated with the help of Equation 22 and the effectively assessed subsample of tariff trees (MANDALLAZ 1997):

$$\frac{1}{n} \hat{V}_x Y(x) = \hat{V}(\hat{Y}) \square \frac{1}{n} \hat{E}_x V(x) = \hat{V}(\hat{Y}) \square \frac{1}{n^2} \sum_{x=1}^n \hat{V}(x) \quad (22)$$

where

$\hat{V}_x(Y) = \frac{1}{n} \hat{V}_x Y(x)$ Variance between the sample plots if all trees would be selected as tariff

trees. $Y(x)$ is the standing volume on the sample plot x and is calculated with a three parameter volume function ($v = f(d_{1,3}, d_7, H)$).

$\hat{V}(\hat{Y}) = \frac{1}{n(n \square 1)} \sum_{x=1}^n (\hat{Y}(x) \square \hat{Y})^2$: Variance between the sample plots if the standing volume is estimated with a subsample of tariff trees (see Equation 19).

$\hat{E}_x V(x) = \frac{1}{n} \sum_{x=1}^n \hat{V}(x)$: Component of the variance between sample plots caused by the

residuals of the tariff trees that are not assessed. It is estimated with the help of the residuals of the assessed tariff trees.

where $\hat{V}(x) = \sum_{i \in S_2(x)} \frac{f_i^2 \square e_i^2 \square (1 \square p_i)}{p_i^2}$ (23)

The standard error of the standing volume estimated with Equation 22 amounts to 0.710% if the d_7 and H are assumed to be measured on all trees. If the tariff trees are selected according to the first NFI, and if the WR technique is applied, the standard error comes to 0.713%. This means that by measuring the d_7 and H on all trees, the variance cannot be significantly reduced. These results are also true for the individual production regions. These standard errors suggest that the sampling fraction of tariff sample trees can be reduced without a significant loss of precision as well.

Selecting a Subsample of Tariff Trees

With the selection of tariff trees, the relationship between cost (c_t) and the obtained precision of the volume estimation (variance between the sample plots s^2) should be optimized. This is the case when $c_t \times s^2$ is minimized.

The cost (c_t) can be divided into fixed costs (c_0) and variable costs (c_1): $c_t = c_0 + c_1$. The fixed costs do not depend on the number of measured tariff trees. They are assumed to be the expenditure for the assessment of the tree data, the individual tree and stand assessments relevant to the standing volume, and the measurements of the sample plot and of the individual trees (phase III of the terrestrial survey in the first NFI without the expenditure for the d_7 and tree height measurements, see ZINGG and BACHOFEN 1988). In addition, many attributes were

assessed in the NFI that are not related to the volume estimation. For that reason not all of the assessment costs were considered for the optimization of the tariff tree selection. The number of trees on which the $d_{1,3}$ was measured remained constant. This is the reason why this expenditure is considered as fixed costs. The variable costs were the result of the number of d_7 and tree height measurements. The expenditures for both of these measurements were determined from time studies on NFI sample plots in different regions of the country, on different stand structures, and in different topographic conditions. It was reasonable to assume overall that it takes five minutes (i.e. 2.5 minutes per person) to measure one tariff tree.

The variance s^2 between sample plots also depends, apart from the number of selected tariff trees, on the selection procedure. Tariff trees can either be selected at random so that each tree has the same probability of being selected, or the selection probability can depend on a measured or estimated tree variable, which is correlated with the target variable, the bole volume.

In the first NFI all trees within the sample plot sector 0–150 gon, as well as all trees with a $d_{1,3} > 60$ cm, were selected as tariff trees. An NFI sample plot consists of two concentric circles with areas of 200 m² and 500 m². Within the smaller area only trees with a $d_{1,3} < 36$ cm are considered. These trees have a lower selection probability than trees with a larger $d_{1,3}$. The selection probability changes suddenly with the limiting diameter of 35 cm. This type of selection procedure is called RS (random selection) in the following.

MANDALLAZ (1991; 1995; 1997) has proven theoretically that a procedure which selects trees proportionally to the prediction error of tariff functions is the most efficient. The probability to select a tree as a tariff tree is greater, the higher the estimated difference between the tariff volume and the bole volume is (i.e. the absolute value of the estimated residual $|\hat{\varepsilon}_i|$). For the optimization, the prediction error was estimated with a regression model. This selection procedure is called PPE (probability proportional to prediction error) in the following.

The $d_{1,3}$ and the basal area $d_{1,3}^2$ are highly correlated with the volume. It is therefore obvious to study the selection proportional to the $d_{1,3}$ (PPS, probability proportional to size) and proportional to $d_{1,3}^2$ (in the following called PPS2).

The tariff tree selection is globally optimized when trees are selected independently of their relationship to a certain sample plot. For the study of the tariff tree selection for the RS procedure, the sample plot sector was continuously reduced from the original 150 gon. For the PPS, PPS2 and PPE procedures, the tariff trees were selected as a subsample from the first NFI tariff trees according to the following rules:

$$t_i = \begin{cases} 1 & \text{for } \gamma \cdot f_i \cdot p_i > u(0) \\ 0 & \text{otherwise} \end{cases}, \quad (24)$$

$$\text{with } p_i \propto \begin{cases} |\hat{\varepsilon}_i| & \text{for the PPE} \\ d_{1,3i} & \text{for the PPS procedure} \\ d_{1,3i}^2 & \text{for the PPS2} \end{cases}$$

i: individual tree

t_i : Binary variable, $t_i = 1$ if a tree was selected as a tariff tree, otherwise $t_i = 0$

γ : Scaling constant that determines the slope of the probability line or the number of selected trees respectively

f_i : Expansion factor: $f_i=20$ for $d_{1,3i}>35$ cm, $f_i=50$ for $d_{1,3i}\leq 35$ cm

$u(0)$: Uniformly distributed random number within the interval [0,1]

The PPS, PPS2 and PPE selection procedures are more efficient than the RS procedure, which can be seen in Figure 5. This figure shows how the sampling fraction of tariff trees affects the

precision of the volume estimation. The graph shows the standard error of standing volume from the first NFI for all four selection procedures of the tariff trees.

The term $c_i \cdot s^2$ reaches its minimum value for the PPS, PPS2 and PPE procedures at approximately 1 tree per sample plot (8,000–12,000 trees for 11,000 sample plots). The PPS2 and PPE procedures give nearly identical results, because the $|\hat{\epsilon}_i|$ increase more or less proportionally to the $d_{1,3}^2$. If the fixed costs are cut in half compared to the above described assumptions, the optimum of the PPS2 procedure is at 5,000 to 8,000 trees; if it is doubled, the optimum is at approximately 13,000 trees (1.2 trees per sample plot).

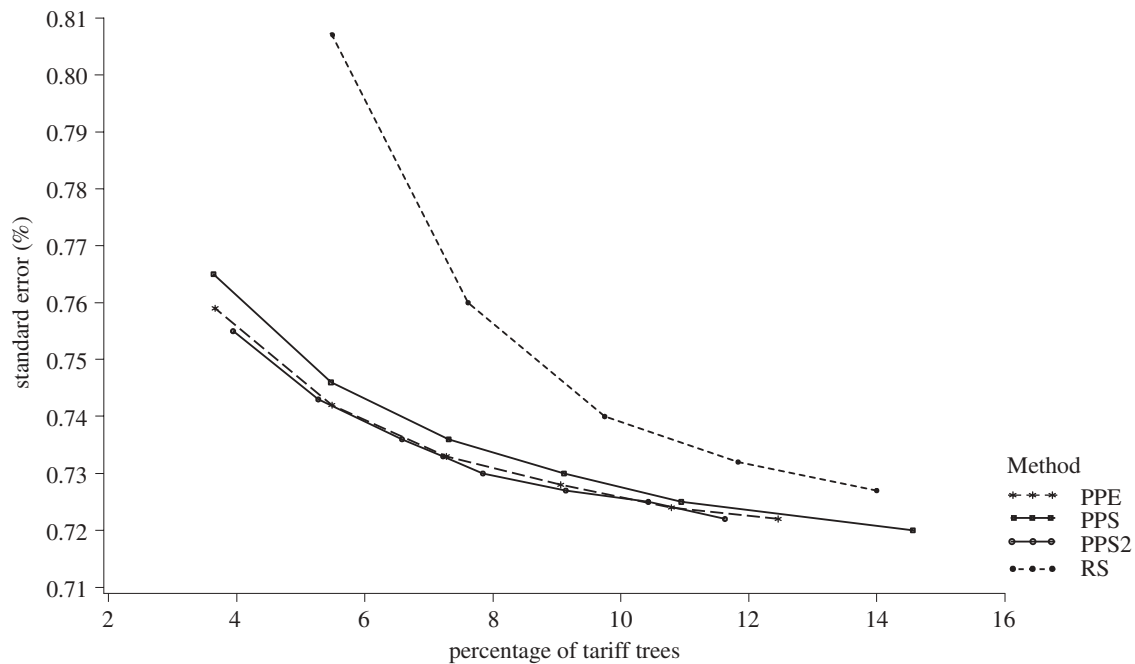


Figure 5. Selection of tariff trees: Standard error of total volume depending on the number of selected tariff trees.

Selection method: PPE: Probability proportional to the prediction error; PPS: Probability proportional to $d_{1,3}$; PPS2: Probability proportional to $d_{1,3}^2$; RS: Selection within a fixed sample plot sector.

Selection of Tariff Trees in the Second NFI

In the second NFI a sufficient number of tariff trees had to be selected for the tariff derivations, taking into account that approximately 2% of the trees are cut every year and that the d_7 or the tree height cannot be measured on approximately 20% of the selected trees. As a consequence, the chosen sampling fraction of two trees was clearly more than the optimum. This was, however, still a reduction by 50% as compared to the first NFI, where on average four tariff trees per sample plot were measured. The selection probability for a sample tree to be selected as a tariff tree is defined as follows:

$$\pi_{\text{Selection}} = 0.000015 d_{1,3}^2 f_i, \text{ for } \pi_{\text{Selection}} \geq 1: \pi_{\text{Selection}} = 1 \quad (25)$$

3.2.5 Standing Volume, Growth and Cut

For the estimation of increment and cut it is necessary to know the state of a tree at the first, as well as at the second inventory date. Changes in the state of individual trees could be determined on the plots that have been assessed at both inventory occasions (i.e. the matched plots). Increment and cut refer, therefore, to the matched sample grid. Conversely, standing volume and change of volume can be estimated by using the matched and unmatched plots.

State of Individual Trees

Changes in the state of individual trees were described in the NFI database with the variables “HISTORY” and “IMMERTOT” as follows:

HISTORY:

- 1 Survivor tree (Tree was recorded at both inventories, see Husch et al., 1972.)
- 2 Ongrowth tree (Tree grew over the caliper-threshold of 35 cm between the two inventories.)
- 3 Ingrowth tree (Tree grew over the caliper-threshold of 12 cm between the two inventories.)
- 4 Cut tree (Tree was removed from the plot between the two inventories. It could be determined that the tree was harvested.)
- 5 Mortality tree (Tree was removed from the plot between the two inventories. It could be determined that the tree has died between the inventories.)
- 6 Cut or mortality tree (Tree was removed from the plot between the two inventories. The cause could not be determined.)
- 8 Tree on a sample plot that was only assessed terrestrially in the first NFI
- 9 Tree on a sample plot that was only assessed terrestrially in the second NFI

IMMERTOT:

- 0 Tree was standing and alive in both inventories
- 1 Tree was already lying or dead in the first inventory
- 2 Tree was standing and alive in the first inventory but lying or dead in the second inventory
- 3 Tree was standing and alive in the first inventory. It was no longer present on the plot in the second inventory

The necessary restrictions when selecting individual trees for the estimation of standing volume, growth, cut or mortality are accomplished with the help of the variables HISTORY and IMMERTOT. These restrictions are presented in Table 13.

Table 13. Selection of trees for the estimation of standing volume, increment and cut.

f1, f2: Expansion factors for the first and second NFI respectively; V1: Volume estimated at the time of the first inventory on the sample plot grid NFI1; V2: Volume estimated at the time of the second inventory on the sample plot grid NFI2 or on the matched grid; V1_1: Volume estimated at the time of the first NFI on the matched grid without back dated ongrowth trees using expansion factors f1; V1_2: Volume estimated at the time of the first NFI on the matched sample plot grid with back dated ongrowth trees using expansion factors f2; V_{sc2}: Volume of the surviving and cut trees at the time of the second NFI; I: Ingrowth; Ggi: Gross growth including ingrowth; Gg: Gross growth without ingrowth; Gd: Net change of volume; CM1.5: Cut and mortality at the middle of the inventory interval.

Plot grid	Variable	Calculation	Inventory	HISTORY	Exp. factor	Restriction
NFI1	V1		LF11	1 4 5 6 8	f1	
NFI2	V2		LF12	1 2 3 9	f2	
Matched	V1_1	V1_1 = V1_2	LF11	1 4 5 6	f1	
NFI1/NFI2	V1_2	V1_1 = V1_2	LF11	1 2 4 5 6	f2	
	Vsc2		LF12	1 2 4 5 6	f2	IMMERTOT # 1
	Vsc2 + I		LF12	1 2 3 4 5 6	f2	IMMERTOT # 1
	V2		LF12	1 2 3	f2	
	Gd	V2-V1_2				
	Gg	Vsc2 - V1_2				
	Ggi	Vsc2 + I - V1_2				
	CM1.5		LF11.5	1 4 5 6	f2	IMMERTOT > 1

Selection of Trees for the Estimation of Standing Volume, Change in Standing Volume, Growth, and Cut

If the codes of the variables HISTORY and IMMERTOT are set according to Table 13, the following volumes are obtained (according to Husch et al. 1972, $CM_{1.5}$ added):

$$\begin{aligned} \text{Standing volume in the first inventory:} & \quad V_1 = V_{s1} + CM_1 \\ \text{Standing volume in the second inventory:} & \quad V_2 = V_{s2} + I \end{aligned}$$

The standing volume estimates are based on individual tree volumes. The expansion to per plot values was accomplished with the WR technique described in Chapter 3.2.3 (Equation 20).

The change in standing volume ($G_d = V_2 - V_1$), refers to the matched sample plot grid.

In the NFI, growth is defined by the following two terms:

G_g: Gross Growth without Ingrowth:

$$\begin{aligned} G_g = V_{s2} - V_{s1} + CM_{1.5} - CM_1 &= V_{sc2} - V_1 & (26) \\ \text{where } V_{sc1} = V_{s1} + CM_1, V_{sc2} = V_{s2} + CM_{1.5} & \end{aligned}$$

G_{gi}: Gross Growth including Ingrowth:

$$G_{gi} = V_{s2} - V_{s1} + CM_{1.5} - CM_1 + I = V_{sc2} - V_{sc1} + I \quad (27)$$

where:

- V_{s2} : Volume of the survivor trees in the second inventory
- V_{s1} : Volume of the survivor trees in the first inventory
- V_{sc1} : Volume of the survivor and the cut trees in the first inventory
- V_{sc2} : Volume of the survivor and the cut trees in the second inventory
- CM_1 : Volume of the cut and mortality trees in the first inventory
- $CM_{1.5}$: Volume of the cut and mortality trees, including their growth up to half of the inventory interval
- I: Ingrowth: Volume of the trees ingrown over the caliper threshold of 12 cm

Ongrowth Trees

The trees which were located within the 500 m² circle of a plot, but outside of the 200 m² circle, and which reached the caliper threshold of 35 cm between the two inventories, were only measured in the second inventory. These trees are called “ongrowth” trees. For the estimation of change in standing volume and increment, the volume of the ongrowth trees at the first occasion is estimated (estimating the $d_{1.3}$ with the help of Equation 15). For both inventories, the expansion factor f_2 , which is valid for the second inventory, is used.

The standing timber that is estimated with the expansion factor f_1 (f_1 is valid for the time of the first NFI) at the first NFI without ongrowth trees ($V1_1$ in Table 13) is the same as the standing volume at the first NFI that is estimated with the ongrowth trees included and with the expansion factor f_2 ($V1_2$ in Table 13).

Definition of the Cut and the Merchantable Timber Volume

A consistent differentiation between cut and natural mortality was not possible in the NFI. The term “cut” comprises the cut and the mortality volume. The term “merchantable timber” denotes the proportion of the merchantable timber volume that was effectively utilized. In the following it is explained how these volumes were determined.

The term $CM_{1.5}$, called “cut”, encompasses the stem volume over the bark of all trees in the matched sample plot grid which were standing and alive during the first inventory, and which

were either lying, dead, or missing in the second inventory (variable IMMERTOT > 1). The “cut” volume includes the growth achieved during one half of the inventory interval. The restriction IMMERTOT > 1 includes the portion of the trees with HISTORY = 1 that had died off between the two inventories but were still present on the sample plot at the second inventory, as well as the proportion of the trees with HISTORY = {4,5,6} that were standing and alive at the first inventory and were missing at the second inventory.

The volume of the merchantable timber comprises the merchantable volume of the trees that have effectively been harvested and utilized between the two inventories.

The merchantable timber volume of a tree consists of the underbark stem and the large branche volume. Stump volume is excluded.

The volume of the merchantable timber was estimated with the help of the proportions presented in Table 14. In table 14, X denotes the merchantable volume of those trees that were standing and alive at the first NFI and that were missing at the second NFI; P(X) is the X- proportion of the total “cut”. P(Y|X) denotes the proportion of the X-volume that was effectively utilized. P(Y|X) is estimated from a subsample of two thirds of all X-trees, for which it was clearly possible to differentiate in the field between effective utilization and natural mortality. The proportion of effectively utilized volume of the “cut” is $P(Z) = P(X) \cdot P(Y | X)$.

The reduction for the stump and bark volumes was carried out differently in Chapter 6 and Chapter 11 of the result volume. In Chapter 6, the overbark stem volume was reduced by a factor depending on the tree species (25% for larch, 10% for beech, 20% for oak and other broadleaf trees excluding ash and maple, and 15% for all other tree species). In Chapter 11, the merchantable stem wood was made up of assortment volumes. The utilized timber volume of both chapters differ by 0.6%.

Table 14. Proportion of effectively utilized overbark timber volume compared to the total cut and mortality volume.

X: Volume of the trees, that were standing and alive in the first and missing in the second NFI.

P(X): Proportion X of the total cut and mortality volume.

P(Y|X): Proportion Y of X that was effectively utilized.

P(Z): Effectively utilized overbark timber volume compared to the total cut and mortality volume.

Region	P(X)	P(Y X)	P(Z) = P(X) * P(Y X)
Jura	0.93	0.99	0.92
Plateau	0.96	0.98	0.94
Prealps	0.9	0.935	0.84
Alps	0.8	0.91	0.73
Southern Alps	0.66	0.75	0.5
Switzerland	0.9	0.95	0.86

3.2.6 Utilizable Timber Volume and Merchantable Assortments

3.2.6.1 Taper Equations

In order to breakdown a bole into merchantable assortments, the stem form must be described with the help of tree variables (in the NFI d_{13} , d_7 , and H) that were measured in the field. The reason for this is because a diameter threshold at any position on the stem is relevant for the assortment (FORSTWIRTSCHAFTL. ZENTRALSTELLE 1976).

Prerequisite for Describing Stem Forms

The prerequisite for modeling stem forms is that there must be a relationship between the measured variables or attributes derived from them and the stem diameter at any arbitrary position. Partial linear dependencies as they are presented in KUBLIN (1987) and KUBLIN and SCHARNAGL (1988), were also found in a preliminary study for the development of NFI taper

equations. The diameter at a certain position on the stem depends linearly on the $d_{1.3}$ and d_7 for trees with the same form ratio $d_7/d_{1.3}$ and the same tree height. There also exists a linear relationship between the form ratio and the diameter for a given $d_{1.3}$ and tree height.

Trees having large form ratios $d_7/d_{1.3}$ in the lower shaft area usually have relatively large form ratios $d_0/d_{1.3}$ in the upper shaft area and vice versa. SLOBODA (1985) called these relationships stochastic rank preservation (in German: stochastische Rangerhaltung).

Figure 6 illustrates this fact using an example of the taper curve of section-wise measured spruce trees that have a $d_{1.3}$ of 35 cm, 40 cm, 45 cm, 50 cm, 55 cm, and 60 cm, a tree height of 35 m and a form ratio ($d_7/d_{1.3}$) between 0.7 and 0.8. The property of rank preservation allows diameters to be estimated at any arbitrary position on the stem when the $d_{1.3}$, an upper diameter and the tree height are known. This is confirmed by the remarkably low standard deviation of differences between the measured and the predicted diameters using the NFI taper model for all main tree species (Table 17).

The stem profile of an individual tree cannot, with satisfaction, be described with the help of the $d_{1.3}$ and the tree height. By including an upper diameter, it is possible to predict the diameter at any arbitrary position of the stem with a sufficient precision. The gain in precision that can be obtained by including other form attributes is relatively small compared to the additional expenditures.

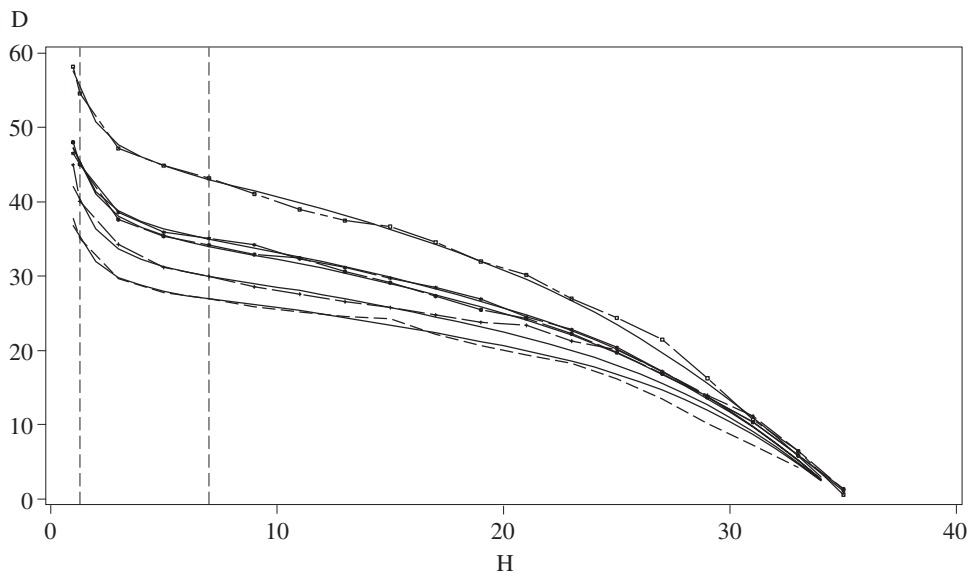


Figure 6. Estimated stem curves (solid lines) and measured diameters, species spruce, $0.7 \leq d_7/d_{1.3} < 0.8$, tree height 35 meters.

Construction of Taper Equations

Based on the relationship between the measured variables and stem form characteristics known from the literature, and based on the results from a preliminary study, cubic interpolation splines (DE BOOR 1978) were chosen to describe stem profiles in the NFI. A Fortran-77 program developed by ENGELN-MÜLLGES and REUTTER (1986) was used for this task. Some diameters at different heights of the stem, which were estimated with the help of a regression model, were used as knots for these splines.

It has been repeatedly shown (BRUCE and MAX 1990; KLEINN 1989; KUBLIN *et al.* 1984; SABOROWSKI 1982; SABOROWSKI *et al.* 1981; SLOBODA 1985; SMALTSCHINSKI 1984; STERBA 1980) that cubic interpolation splines are very suitable to describe stem profiles. A cubic spline consists of polynomial segments of order three that are twice continuously differentiable. The function values and the first and second derivatives of two adjacent polynomials are the same at the knots (i.e. the intersection points between two polynomials), so that the polynomials that describe one stem profile result in a smooth curve.

The NFI taper equations are constructed as follows:

1. Estimation of upper stem diameters with the help of a regression model

Upper diameters at several locations (i) on the stem are estimated with the help of a regression model. This model was developed with the data stemming from section-wise measured sample trees. The diameters d_{ji} in 1 meter height, at 5%, 10%, 20%, 30%, 50%, 70%, and 80% of the stem length are estimated with the following regression model, depending on the three variables $d_{1.3}$, d_7 and H of the tree i:

$$\hat{d}_{ji} \frac{d_{7i}}{d_{1.3i}} = b_{0j} + b_{1j} \cdot H_i + b_{2j} \cdot d_{7i} + b_{3j} \cdot d_{7i} H_i + b_{4j} \cdot d_{1.3i} H_i \quad (28)$$

$$\text{Weight: } w_i = \left(\frac{d_{1.3i}}{d_{7i}} \right)^2$$

In order to obtain unbiased estimates of the d_{ji} , the squared errors are weighted with w_i so that:

$$\sum_{i=1}^n \left[(d_{ji} - \hat{d}_{ji}) \cdot \frac{d_{7i}}{d_{1.3i}} \right]^2 \cdot W_i = \text{minimum}$$

Substituting W_i in this equation

$$\sum_{i=1}^n \left[(d_{ji} - \hat{d}_{ji}) \cdot \frac{d_{7i}}{d_{1.3i}} \right]^2 \cdot \left(\frac{d_{1.3i}}{d_{7i}} \right)^2 = \sum_{i=1}^n [d_{ji} - \hat{d}_{ji}]^2 = \text{minimum}$$

The coefficients for equation 28 can be found in Table 15.

Especially for broadleaf trees, the diameter at the lower stem areas can be predicted more precisely than for the upper stem areas, since the position of the crown base influences the stem form. For coniferous trees, the coefficient of determination (R^2) amounts to 99.5–99.8% for the $d_{0.3}$, 98.1–99.2% for the $d_{0.5}$ and 90.4–95.5% for the $d_{0.8}$. For beech, the R^2 comes to 99.4% for the $d_{0.3}$, 95.9% for the $d_{0.5}$ and 78.7% for the $d_{0.8}$. The predicted diameters uniformly decrease from the stem base to the top for all trees.

Table 15. Coefficients of the functions that predict upper stem diameters at fixed locations.

Species	Location	b0	b1	b2	b3	b4
Spruce	1m	-0.0322735	0	1.01822722	0	0.00070241
	5%	0.6694153	-0.03912357	1.10731936	0.00183994	-0.00460934
	10%	1.98212707	-0.08931625	1.01658607	0.01297247	-0.01315235
	20%	2.32979631	-0.08534539	0.90240645	0.02113534	-0.01876776
	30%	1.94286823	-0.05605474	0.83535552	0.02046769	-0.01817887
	50%	1.2168963	0.00958755	0.65084147	0.01887258	-0.0163289
	70%	0.1701299	0.11005913	0.37702981	0.01457552	-0.01197956
	80%	-0.35434648	0.14221273	0.23483065	0.01028162	-0.00846602
Fir	1m	-0.61671376	0.01348875	1.05035114	-0.0020169	0.00156697
	5%	0.38173759	-0.01419189	1.08620965	0.00221964	-0.00474542
	10%	2.26735806	-0.07040213	0.95987737	0.0144543	-0.01352572
	20%	2.96815848	-0.09566062	0.87055081	0.02176994	-0.0186884
	30%	2.73444057	-0.08702372	0.81440997	0.02087228	-0.01768526
	50%	1.1004138	0.01780012	0.66899645	0.01685419	-0.01450547
	70%	-1.5623776	0.22157046	0.42897272	0.00991785	-0.00938154
	80%	-2.1747944	0.27151403	0.26857728	0.00707074	-0.00680849
Scotch Pine	1m	0.33030733	0	1.02457213	0.00379692	-0.00300232
	5%	0.63361645	-0.02878408	1.11662853	-0.00140075	-0.00226452
	10%	1.64052474	-0.07434391	1.02397656	0.00732716	-0.00865593
	20%	2.52197576	-0.10863366	0.89019656	0.02174408	-0.01854897
	30%	2.25426126	-0.0958468	0.83243483	0.02315113	-0.01978661
	50%	1.77930748	-0.07293812	0.7132659	0.02046024	-0.01785395
	70%	0.59456933	-0.04502022	0.64238423	0.01239497	-0.01308449
	80%	-0.21832582	0.02805848	0.49054864	0.00834929	-0.01011157
Larch	1m	-0.66468376	0.05510109	1.02942657	-0.00196514	0.00125772
	5%	1.2846204	-0.05446032	1.09672856	0.00096711	-0.00355392
	10%	2.63373613	-0.10524832	0.97468442	0.01231533	-0.01152565
	20%	2.06671786	-0.07194527	0.89571863	0.01959724	-0.01754426
	30%	1.47027445	-0.0459033	0.83956546	0.01894159	-0.01717868
	50%	0.71770215	0.02104711	0.6494292	0.01692222	-0.01490832
	70%	-0.29833773	0.12124127	0.40120178	0.01198215	-0.01057149
	80%	-0.61502206	0.15138558	0.25490171	0.00875077	-0.00784795
Beech	1m	-0.10826337	0	1.03912008	-0.00089265	0.0004035
	5%	0.59974957	-0.01922579	1.04605079	0.00017884	-0.00176683
	10%	1.30630982	-0.03507829	0.98725438	0.00997312	-0.01022303
	20%	1.69247139	-0.05899471	0.9332602	0.01975506	-0.0184943
	30%	1.686064	-0.06477597	0.86990666	0.02075929	-0.01893105
	50%	0.6492604	-0.00148954	0.6994524	0.01686741	-0.01543733
	70%	-1.2748044	0.13125174	0.43584272	0.00958976	-0.00995488
	80%	-0.72813153	0.117301	0.22970511	0.00731678	-0.00673901
Oak	1m	-0.39968038	0	1.08216906	-0.0013725	0
	5%	0.28164521	-0.01204597	1.09435189	-0.00087433	-0.00222605
	10%	1.49493408	-0.05652448	1.0137068	0.01020512	-0.01101364
	20%	1.48586392	-0.06749376	0.98738503	0.0208354	-0.02066637
	30%	1.75309861	-0.07523583	0.89548081	0.02410906	-0.02224103
	50%	0.50951594	-0.00726158	0.74923503	0.01699877	-0.0161029
	70%	-0.80354124	0.11033778	0.48804128	0.01151507	-0.01212845
	80%	-0.62629098	0.11335155	0.29778415	0.00792077	-0.00831111

2. Interpolation between estimated diameters with the help of cubic splines

Between two neighboring estimated diameters, a taper curve is interpolated with a cubic spline. Several spline segments describe a stem profile with a continuous smooth curve. The goodness-of-fit of these splines depends mainly on the number and the location of the knots. The locations according to Hohenadel (d0.1, d0.3, d0.5, d0.7, d0.9 HRADETZKY 1981; KUBLIN and SCHARNAGL 1988; SABOROWSKI 1982) are suitable as knots. Interpolation for polynomial segments with neighboring polynomials is unproblematic, since the first and second derivatives are given at the endpoints. They are defined by the equation system of the continuity conditions (same function values and first and second derivatives of neighboring polynomial segments). Nevertheless, the end segments have to be treated separately. In order to determine the polynomial coefficients, either the first (slope), second (curvature), or the third derivative of the curve must be known in addition to the diameter. If the curvature is 0 at both end points ($f''(x)=0$), it is called a natural cubic spline (DE BOOR 1978; HRADETZKY 1981). In order to

obtain a slightly better fit around the base of the stem, and to reduce the tendency of the spline to underestimate in the upper parts of the stem, the splines are tied in at both of their endpoints (i.e. the curvature is given). SABORWOSKI (1982) had obtained the best fit in the area between 1.3 m and 7 m by setting the starting curvature individually for each tree as well.

For the starting values at the stem base, the shaft curvature is estimated. In order to accomplish this, a polynomial with the form $y = a + b x^2 + c x^3$ is fitted to the measurement points of the section-wise measured trees at 1.0 m, 1.3 m, and 3.0 m. The curvature, that is the second derivative of this polynomial at 1 m is the target parameter in a regression model which estimates the curvature at 1 meter with the $d_{1.3}$, $d_{1.0}$ and d_7 .

$$f''(1.0) = b_0 + b_1 \cdot d_{1.3}^2 + b_2 \cdot d_7^2 + b_3 \cdot \frac{d_{1.3}}{d_{1.0}} + b_4 \cdot d_{1.3} d_{1.0} \quad (29)$$

The coefficients for the curvature model at the stem basis can be found in Table 16. An estimated curvature at the bottom is given for each tree; the curvature at the top is kept constant (-0.001).

Table 16. Coefficients of the function that estimates the curvature at the stem basis.

	b0	b1	b2	b3	b4
Spruce	0.668756	-5.63213E	9.09932E	-0.679303	4.87134E
Fir	0.825355	-4.72139E	6.47683E	-0.836509	4.15204E
Scotch Pine	0.595389	-7.60644E	9.80589E	-0.612327	6.81946E
Larch	0.894741	-4.34839E	6.57241E	-0.920326	3.83439E
Beech	0.643240	-6.54322E	7.60462E	-0.651962	5.87191E
Oak	0.691773	-6.46014E	7.9513E	-0.701271	5.76082E

3. Adjusting the taper curve

A taper curve formed with the estimated diameters do not go exactly through the measured $d_{1.3}$ and d_7 . From the statistical point of view this is not necessary as long as the $d_{1.3}$ and d_7 are unbiasedly estimated. If the objective is, however, to compare individual tree volumes estimated with bole volume functions with volumes estimated with taper equations, the individual tree curves have to go through both measurement points. For this purpose the two measurement points, $d_{1.3}$ and d_7 , could have been used as additional knots for the spline interpolation. However, using measured and estimated diameters at the same time as knots results very often in curves that do not uniformly decrease or in a strong oscillation of the spline. Only the positions mentioned in the first step are therefore used as knot diameters. With the help of the difference between the measured and the estimated $d_{1.3}$ and d_7 , the curve of each tree is shifted so that it intersects with both measured diameters. The adjusting function is linear. The correction factor is proportional to the inverse distance between point h_j on the shaft and the height 1.3 m or 7 m.

$$da(h) = \hat{d}(h) + \frac{(h - a)}{b} \cdot (d_7 - \hat{d}_7) + \left(1 - \frac{(h - a)}{b}\right) \cdot (d_{1.3} - \hat{d}_{1.3}) \quad (30)$$

for $h \leq 7$ meters

$$da(h) = \hat{d}(h) + \left(1 - \frac{h - c}{H - c}\right) \cdot (d_7 - \hat{d}_7) \quad (31)$$

for $h \geq 7$ meters

$\hat{d}(h)$, \hat{d}_7 , $\hat{d}_{1.3}$: Diameter estimated with the help of a spline-Interpolation in centimeters

h :	Arbitrary location on the shaft in meters
$da(h)$:	Adjusted diameter in centimeters
H :	Tree height in meters
a :	1.3 meters
b :	5.7 meters (distance between $d_{1.3}$ and d_7)
c :	7 meters

The 99% percentiles of this adjustment are for section-wise measured spruce at ± 0.44 cm for the $d_{1.3}$, at ± 1.32 cm for the d_7 , and for spruce tariff trees assessed during the first NFI, at ± 1.33 cm (for the $d_{1.3}$) and ± 3.6 cm (for the d_7). The corresponding 95% percentile of the correction is at 1 mm and 3 mm for the section-wise measured spruce and at 0.6 cm and 2.0 cm for the NFI tariff sample trees. On average, a stem curve of the section-wise measured spruce is adjusted by an absolute value of 1 mm for the $d_{1.3}$ and 3 mm for the d_7 . For spruce from the first NFI they were adjusted by 2 mm ($d_{1.3}$) and 6 mm (d_7). These corrections do not create any systematic error, since the average adjustments did not deviate from zero.

Validation of the Taper Equation

Table 17 shows the average deviations (\bar{x}) of measured from estimated diameters in the tree height class 30–36 m, as well as the standard deviation of the differences (s_x) along the stem. The standard deviations of the residuals of 1.0–1.5 cm for conifers indicates that the stem profiles can be described very precisely with the chosen method. The standard deviation of the residuals, especially in the upper shaft area, with approximately 2.5 cm for beech and approximately 3.5 cm for oak are higher, since the crown base and the crown volume have a larger influence on the shaft form of broadleaf trees than of coniferous trees. For illustration, Figure 7 shows the estimated stem curve and the variation width of the shaft forms for all section-wise measured firs with a height of 34 m, a d_7 between 44 and 45 cm and a $d_{1.3}$ between 55 and 56 cm.

A stem curve must uniformly decrease from the stem base to the top. This is the case for 98.5% of all calculated stem curves for tariff sample trees in the first NFI. Especially for trees with a very large form ratio ($d_{1.3}/d_7 > 0.9$), it is possible that a spline slightly increases between two knots. In these cases it is assumed that the corresponding shaft forms are cylindrical in these areas. It is rare (0.5% of all trees) that the splines oscillate so strongly that no stem profile can be calculated.

All calculated stem curves were checked visually on the computer screen for their behavior when the d_7 changed and the $d_{1.3}$ and H stayed the same. These gradual changes are harmonic for all possible $d_{1.3}$ -height combinations; that is, stem curves with the same tree height and the same $d_{1.3}$ do not intersect with each other when the d_7 is changed.

The volumes estimated with the taper functions are only slightly less precise (Table 18, standard deviation of the residuals) than those estimated with bole volume functions. The taper functions have a slight tendency to underestimate the volume, which is due to the way the stem curves were predicted in the upper stem areas.

The volumes of the NFI1 tariff sample trees are estimated to be slightly smaller by the taper functions than by the bole volume functions (the difference ranging from 0.2% for spruce up to 1.4% for fir and larch, see Table 19). The average difference between volumes estimated by the bole volume functions and those estimated by the taper functions, as well as the standard deviation of these differences, are very low (KAUFMANN 1993). This demonstrates that the bole volume functions as well as the taper functions are stable.

Table 17. Average deviations (\bar{x}) between predicted and measured stem diameters and standard deviations of the residuals (s_x) in cm.

Location	Fir 1692		Spruce 2517		Scotch Pine 207		Larch 438		Beech 1179		Oak 72	
	\bar{x}	s_x	\bar{x}	s_x	\bar{x}	s_x	\bar{x}	s_x	\bar{x}	s_x	\bar{x}	s_x
1 m	-0.25	1.4	-0.21	1.21	-0.16	0.8	-0.50	1.3	-0.14	0.7	-0.03	0.9
3 m	-0.16	1.1	-0.16	0.92	-0.04	1.1	-0.08	0.9	-0.12	0.8	-0.25	1.0
5 m	0.01	0.7	-0.02	0.60	-0.13	1.3	-0.02	0.7	-0.05	0.7	-0.20	1.3
7 m	0.00		0.00		0.00		0.00		0.00		0.00	
9 m	-0.01	0.8	0.04	0.56	-0.01	1.0	-0.06	0.7	0.03	1.0	-0.17	1.2
11 m	0.00	1.0	0.05	0.69	0.03	1.2	-0.02	0.9	0.08	1.4	0.16	1.3
13 m	-0.01	1.2	0.03	0.83	0.02	1.5	-0.07	1.1	0.13	1.8	-0.39	1.6
15 m	-0.02	1.5	0.02	0.98	-0.10	1.5	-0.11	1.2	0.11	2.0	-0.28	2.5
17 m	-0.05	1.8	0.00	1.16	0.01	1.6	-0.09	1.3	0.12	2.3	0.02	3.4
19 m	-0.11	2.0	-0.01	1.29	0.22	1.7	-0.06	1.5	0.28	2.6	-0.09	3.5
21 m	-0.17	2.3	-0.04	1.41	0.21	1.7	-0.02	1.5	0.29	2.5	-0.27	3.6
23 m	-0.23	2.5	-0.12	1.53	0.18	1.6	-0.04	1.5	0.10	2.4	-0.42	3.5
25 m	-0.24	2.5	-0.16	1.52	0.01	1.6	-0.07	1.5	0.22	2.1	-0.70	3.2
27 m	-0.32	2.3	-0.17	1.42	0.38	1.7	0.01	1.5	0.58	1.6	-0.28	2.8
29 m	-0.44	1.9	-0.20	1.20	0.31	1.4	0.09	1.1	0.70	1.2	0.06	2.6
31 m	-0.48	1.6	-0.19	0.95	0.24	1.1	0.20	0.8	0.69	0.9	0.19	2.0
33 m	-0.49	1.3	-0.22	0.73	0.11	0.9	0.25	0.6	0.55	0.6	0.11	1.5
35 m	-0.44	0.7	-0.21	0.39	0.23	0.4	0.12	0.3	0.29	0.3	0.06	1.0

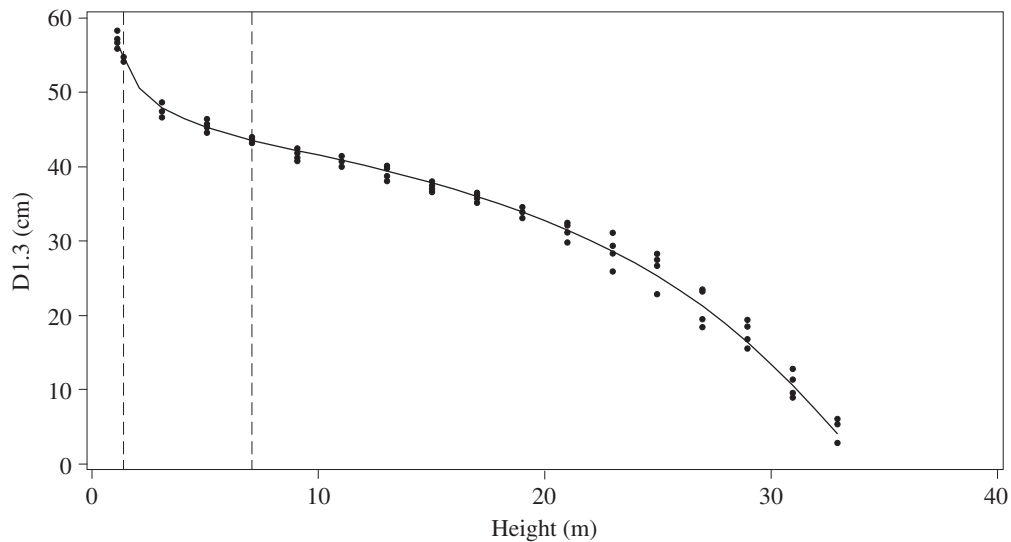


Figure 7. Predicted stem curve (solid line) and measured diameters, species fir, all section-wise measured trees having $d_{1.3} = 55\text{--}56$ cm, $d_7 = 44\text{--}45$ cm, tree height = 34 meters.

Table 18. Goodness-of-fit of the taper equations. ar: average residual ($\Sigma(\hat{x}_i - x_i) / n$) and standard deviation of the residuals (in parenthesis) in percentage of the mean. p_i: Probability of the t-distribution. n: Number of trees. R²: Coefficient of determination.

		d1.3 (cm)									
		12.-23	24-35	36-47	48-59	60-71	72-83	84-95	96-107	total	
Spruce (R ² : 99.6%)	ar (sr)	-0.4 (3.7)	-0.8 (3.7)	-1.0 (4.0)	-1.1 (4.5)	-0.5 (5.5)	0.8 (5.2)	-0.4 (5.2)	-0.8 (1.7)	-0.8 (7.0)	
	p _i	0	0	0	0	0.17	0.25	0.8	0.52	0	
	n	6617	3521	1780	779	253	60	11	3	13024	
Fir (R ² : 99.6%)	ar (sr)	0.3 (4.6)	-1.0 (4.6)	-1.0 (4.9)	-1.2 (4.9)	-0.7 (4.9)	-0.7 (5.1)	-0.3 (4.4)	-0.8 (4.1)	-0.9 (7.2)	
	p _i	0.02	0	0	0	0	0.08	0.57	0.42	0	
	n	1628	2199	1495	848	423	158	67	19	6848	
Scotch Pine (R ² : 99.2%)	ar (sr)	0.0 (4.7)	-1.1 (3.9)	-1.2 (5.0)	2.0 (8.1)	2.0 (5.7)	4.7			-0.8 (6.4)	
	p _i	0.86	0	0	0.17	0.7				0	
	n	487	789	334	31	2	1			1644	
Larch (R ² : 99.4%)	ar (sr)	-0.4 (3.7)	-1.4 (3.9)	-1.0 (4.2)	-0.7 (4.5)	-1.0 (4.2)	-2.1 (6.3)	-0.9 (12.3)	-2.4	-1.00 (6.3)	
	p _i	0.06	0	0	0.01	0.02	0.1	0.88		0	
	n	303	454	494	232	102	26	5	1	1617	
Beech (R ² : 98.7%)	ar (sr)	-0.4	-0.8	-0.9	-0.9	-2.2	-2.5			-0.9 (11.5)	
	p _i	0	0	0	0.14	0.17	0.5			0	
	n	2515	2188	863	244	46	7			5863	
Oak (R ² : 99.5%)	ar (sr)	0	-0.9	-1.7	0.5	-0.7	-1.9	-3.1		-0.8 (8.3)	
	p _i	0.98	0	0	0.48	0.48	0.1	0.35		0	
	n	621	578	254	98	37	9	2		1599	

Table 19. Volumes estimated with taper equations (\mathcal{S}_{1i}) compared with volumes estimated with bole volume functions (\mathcal{S}_{2i}): ar: average difference $\Sigma(\mathcal{S}_{1i} - \mathcal{S}_{2i}) / n$ and standard deviation of the differences (in parenthesis) in percentage of the mean. n: Number of trees.

		d1.3 (cm)									
		12.-23	24-35	36-47	48-59	60-71	72-83	84-95	96-107	total	
Spruce	ar (sr)	-0.7 (2.5)	-1.1 (1.2)	-0.9 (1.0)	-0.3 (1.6)	0.7 (2.4)	1.3 (3.8)	2.5 (6.3)	1.0 (12.2)	-0.2 (4.0)	
	n	4993	3680	6020	2682	1738	430	70	24	19640	
Fir	ar (sr)	0.1 (8.9)	-1.2 (1.2)	-1.4 (0.8)	-1.4 (1.6)	-1.3 (1.0)	-1.4 (1.1)	-1.4 (1.3)	-2.7 (2.5)	-1.4 (3.3)	
	n	1778	1095	1612	647	497	154	28	6	5822	
Scotch Pine	ar (sr)	-1.7 (3.5)	-2.5 (1.7)	-1.5 (1.5)	0.0 (1.9)	2.2 (3.7)	5.7 (2.0)	12.4		-0.6 (4.1)	
	n	294	279	547	197	59	4	1		1381	
Larch	ar (sr)	5.4 (5.8)	-0.1 (2.3)	-1.1 (1.1)	-1.3 (0.8)	-1.8 (1.0)	-2.2 (1.3)	-3.5 (1.8)	4.0 (2.2)	-1.4 (3.0)	
	n	482	372	608	312	282	84	23	7	2172	
Beech	ar (sr)	-0.1 (2.6)	-1.0 (1.3)	-1.1 (1.3)	-1.2 (2.3)	-1.4 (2.8)	-2.7 (3.7)	-7.9 (4.6)	-12.6	-1.2 (4.5)	
	n	3522	1852	1875	607	293	67	10	1	8227	
Oak	ar (sr)	-0.7 (5.6)	-2.8 (2.7)	-0.8 (1.8)	0.4 (2.8)	1.0 (3.6)	0.9 (3.1)	1.4 (5.9)	5.5 (5.1)	0.1 (6.0)	
	n	402	186	182	82	68	28	9	3	960	

3.2.6.2 Assortments

Stem volumes are calculated with the help of rotational integrals, through which the spline function ($f(x)$), the adjustment function ($g(x)$) and the bark reduction function ($r(x)$), ALTHERR *et al.* 1978) are overlaid with each other.

$$V = p \sum_{i=1}^q \int_{x=h_{i-1}}^{h_i} [f(x) + g(x) - r(x)]^2 \cdot dx \quad (32)$$

where

- x: Height on the shaft
- q: Number of integration limits
- h_i: Integration limits at the following locations:
 - a) Knots (j) of the splines
 - b) Intersection point of the of the adjustment functions (7 meters)
 - c) Limits (k) of the stem sections for the bark reduction models

(0.33 H, 0.66 H; H= tree height)

d) Assortment limits for merchantable assortments

h_0 : Basis (x=1 m)

The section from the stem base up to 1 meter in height is assumed to be cylindrical with a diameter of d_{1m} .

Functions:

$$f(x) = b_{0j} + b_{1j} \cdot rx_j + b_{2j} \cdot rx_j^2 + b_{3j} \cdot rx_j^3 \quad (33)$$

j: Knots of the spline function

$$rx_j = x_j - x_{j-1}$$

$$g(x) = c_1 \cdot x + c_2 \quad (34)$$

c_2 : $da_i - d_i$ at the location h_{i-1}

$$c_1 : \begin{cases} \left((d_7 - \hat{d}_7) - (d_{1.3} - \hat{d}_{1.3}) \right) / 5.7 & \text{for } h_{i-1} \leq 7 \\ -(d_7 - \hat{d}_7) / (H - 7) & \text{for } h_{i-1} > 7 \end{cases}$$

$$r(x) = b_{0k} + b_{1k} \cdot d_{mk} - b_{2k} \cdot dm_k^2 \quad (35)$$

$$\text{where } k : \begin{cases} 1 & \text{for } x = 0 \leq x = 0.33 \cdot H \\ 2 & \text{for } x = 0.33 \cdot H \leq h_{i-1} = 0.66 \cdot H \\ 3 & \text{for } h_{i-1} > x = 0.66 \cdot H \end{cases}$$

dm_1 : Diameter over bark for $x=0.25 H$

dm_2 : Diameter over bark for $x=0.50 H$

dm_3 : Diameter over bark for $x=0.75 H$

In the NFI, the input variable d_7 and tree height (H) are only known for the tariff trees. The d_7 and H of the remaining trees are predicted with the NFI tariff functions (see Chapter 3.2.2.2). Table 20 shows assortment proportions of the spruce tariff trees of the first NFI. These proportions are calculated in three different ways (columns A to C). The percentages presented in column A are based on stem profiles that have been calculated with the help of measured d_7 and H. Those in column B are based on stem profiles that have been constructed with the help of estimated d_7 and H, and those in column C are based on assortment tariffs (see Chapter 3.3.1.4). The differences between the three estimation methods are very small.

Table 20. Estimated proportions of assortments with different methods using the spruce tariff trees of the first NFI (100%: total bole volume under bark).

Estimation methods:

A: Taper equations, d7 and h measured. B: Taper equations, d7 and h estimated. C: Assortment tariff.

Region	Assortment	A (%)	B (%)	C (%)	
Jura	Long stemwood	1. Class	17.9 (1.3)	15.7	18.7
		2. Class	23.6 (1.3)	25.4	23.1
		3. Class	26.6 (1.2)	27.9	26.3
		4. Class	12.0 (0.8)	12.7	12.3
		5. Class	9.8 (0.7)	8.1	10.5
		Total	89.8 (0.4)	89.7	90.9
	Short stemwood	middle diam. \geq 30 cm	53.9 (1.2)	54.7	52.9
		middle diam. < 30 cm	27.9 (0.8)	27.0	28.5
		Total	81.9 (0.6)	81.6	81.4
Plateau	Long stemwood	1. Class	25.6 (1.2)	25.9	25.3
		2. Class	31.9 (1.0)	32.9	31.3
		3. Class	20.4 (0.9)	19.4	21.3
		4. Class	7.3 (0.5)	6.8	7.6
		5. Class	5.0 (0.4)	4.7	5.5
		Total	90.2 (0.3)	89.7	91.0
	Short stemwood	middle diam. \geq 30 cm	51.6 (0.9)	51.4	52.5
		middle diam. < 30 cm	30.8 (0.6)	30.1	28.9
		Total	82.4 (0.5)	81.5	81.4
Prealps	Long stemwood	1. Class	21.3 (1.0)	19.5	20.2
		2. Class	24.5 (0.9)	26.4	26.4
		3. Class	23.3 (0.8)	24.4	20.9
		4. Class	10.7 (0.6)	11.5	12.0
		5. Class	9.1 (0.5)	7.3	9.6
		Total	88.9 (0.3)	89.1	89.1
	Short stemwood	middle diam. \geq 30 cm	53.5 (0.8)	53.5	52.8
		middle diam. < 30 cm	28.3 (0.6)	28.0	28.7
		Short stemwood	81.8 (0.4)	81.5	81.5
Alps	Long stemwood	1. Class	16.9 (0.8)	13.4	16.7
		2. Class	18.5 (0.7)	20.0	19.1
		3. Class	23.7 (0.7)	26.7	22.9
		4. Class	15.1 (0.5)	16.3	14.7
		5. Class	12.7 (0.5)	10.9	13.9
		Total	86.9 (0.3)	87.4	87.3
	Short stemwood	middle diam. \geq 30 cm	52.4 (0.8)	52.0	52.5
		middle diam. < 30 cm	27.9 (0.5)	27.9	28.4
		Total	80.3 (0.4)	79.9	80.9
Southern Alps	Long stemwood	1. Class	12.6 (2.0)	8.7	14.9
		2. Class	17.7 (1.7)	15.9	16.1
		3. Class	25.8 (1.9)	31.0	24.3
		4. Class	16.1 (1.6)	18.6	16.7
		5. Class	13.7 (1.3)	11.8	13.6
		Total	85.9 (0.9)	86.1	85.6
	Short stemwood	middle diam. \geq 30 cm	57.2 (1.8)	56.1	56.5
		middle diam. < 30 cm	24.0 (1.1)	24.4	25.8
		Total	81.2 (0.9)	80.5	82.3

3.2.7 Literature

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